Dynamic General Equilibrium Analysis of the Term Structure of Interest Rates

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Abstract

In this paper, I investigate the determinants of bond risk-premia using a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model. The model features adjustment cost in capital and habit formation preferences with three shocks. The contributions of the model shocks to risk-premia are analysed. The model is solved by perturbation method and the shocks are estimated by Simulated Method of Moments (SMM) using U.S. quarterly data. The implied second-order approximate solution delivers positive risk-premia. Results suggest that when the capital stock is fixed, a higher habit formation parameter significantly increases the risk premium. However when the capital stock is allowed to vary, increases in habit strength decrease risk premium. Moreover, monetary policy has a significant impact on interest rates premia. Especially, an agressive monetary policy leads to decreases in risk premia. This is because fighting against inflation reduces inflation volatility and then decreases the inflation risk premium. In terms of contribution of the three shocks in the benchmark model, preferences shocks contribute far more to the risk premiums followed by productivity shocks with a less important role for monetary policy shocks.

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1 Introduction

The goal of this work is to investigate the determinants of the term structure of interest rates in a New-keynesian Dynamic Stochastic General Equilibrium (DSGE) model with habit formation preferences and adjustment cost in capital stock. The model features three shocks: productivity shock, preferences shock and monetary policy shock. I ask how changes in macroeconomic structural parameters such as preferences, technology or monetary policy parameters affect the term structure of interest rates. The contribution of the three shocks to the size of risk premia is also studied.

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New keynesian models are known to replicate many empirical business cycle facts[1] and are increasingly used in many central banks for policy analysis. It is then important to understand how interest rates behave in this framework because changes in central bank instrument rate are intended to pass-through the term structure of interest rates and affect the real economy. The relationship between interest rates that only differ in maturities is an important area of research because economists believe that important economic facts can be inferred from this relationship. In fact empirical works have found the yield curve to have economic growth prediction power over a long period of time (see for example, Harvey(1991)). Second, the term structure of interest rates contains important implications for market expectations about monetary policy and inflation forecast.

Long-term interest rates can be explained as market expectations about future short-term interest rates (the traditional expectation hypothesis theory) and risk premia. As for the market expectations about future short-term rates, it means that current long-term interest rates reflect the investors anticipations about the future monetary policy stances because short-term rates are controlled by monetary policy authorties. Thus, long-term interest rates reflect future expectations of inflation and output in the New-kenesian environment. The risk premium component compensates the investors for the risk born by holding a long-term debt instead of rolling over short-term instruments. In this model, the risk arises for two reasons. First, an investor fears future capital losses because there is uncertainty about the bond future prices. Even with risk-free bonds, a capital loss can happen if the holder want to resell the bond before maturity time to offset a bad income shock for example. If it happens that the resell price is very low, she will suffer a consumption fall. Second, inflation can erode the value of the bond even at maturity time because the bonds are nominal. Risk premia are then as important as the expectations part for the central bank because they affect the long-term interest rates as well. Unfortunately risk premia are unobserved and can have undesirable impacts on monetary policy. For example, a tightening monetary policy effect can be undermined by a decline in the risk premium component even if the market correctly anticipates the future monetary policy actions as it recently happened in 2004 in the U.S. economy[2]. Kurmann and Otrok (2011) find in VAR framework a weak long-term interest rates response

[1] See Smets and Wouters (2003); Christiano, Eichenbaum and Evans (2005).

[2] See Cochrane and Backus (2007), Rudebusch et al (2007) for this issue called the "Greenspan Conundrum" in Finance literature

to a news productivity shock because the responses of the term premium and the expectations part offset each other. It is then important- at least for central bankers- to understand the economic determinants of risk premia.

It is challenging to study the term structure of interest rates in a DSGE model. Especially, risk premia are difficult to compute because DSGE models are non-linear systems and an analytical solution is unavailable for the general case. Numerical methods such as value function iteration (VFI) or policy function iteration (PFI) are computationally infeasible because of the large number of state variables. Moreover, previous work has found standard Real Business Cycle (RBC) models to mismatch simultaneously business cycle variables and asset prices[3]. In exchange economy frameworks, some of these puzzles have been solved by using either habit formation preferences (see Campbell and Cochrane(1999), Wachter (2005), Piazzesi and Schneider (2007)) or recursive preferences (Gallmeyer *et al* (2008)). This is because with these preferences, risk aversion becomes countercyclical (instead of being constant) and resources can only be allocated intertemporarily through financial assets. Thus a risk averse investor will require a larger compensation to hold a long-term bond instead of rolling over short-term bonds. In production economy frameworks where consumption, output, and investment are endogenous, there are other channels available for consumption smoothing than the financial assets. The agent could either increase his labour or uses investment every time to offset unexpected bad income shocks given that the cost of adjusting these variables are low. Thus, the increasing effect of habit formation preferences- on bond risk premia size- will tend to be weakened in production economies. There are also evidences that the bond premium puzzle remains unsolved in New keynesian models even with habit formation preferences and real rigidities. Rudebusch et al (2008) find that the volatility of risk premia is insignificant in a New keynesian model with real rigidities such as capital adjustment cost and adjustment cost in labour market.

Therefore, I focus in this work on the structural determinants of the size of bond risk premia. In early studies of the term structure of interest rates in production economies, higher habit strength parameter increases the size of risk premia. However, the capital input factor in the production function is fixed in these papers (Rudebusch *et al* (2008), Ravenna and Seppala (2006)). I compare the habit forma-

[3]Donaldson, Merha and Prescott (1990) found that a RBC model with full depreciation of capital cannot replicate bond risk premium consistent with the data (bond premium puzzle). see also Den Haan (1995)

tion preferences effect on the size of bond risk premia when the capital stock is fixed and when the capital stock can be adjusted costlessly. I also ask to what extent each shock contributes to the size of risk premia and if the agent prices the risk involved in these shocks in the same way. That is, I decompose the contribution of each shock to two multiplicative terms: first, the size of the volatility of the shock that captures the quantity of risk it brings with. Second, a constant function of structural parameters that can be interpreted as price of the associated risk. Thus, macroeconomic factors of risk premium operate through the volatilities of exogenous shocks (quantity of risk) and these scaling coefficients (price of risk). It is well known that increasing the size of the volatilities of the shocks will magnify the size of risk premia. The ability of a DSGE model to generate a sizeable risk premium will depends on the shock volatilities. Rudebusch and Swanson (2008) find in a calibrated DSGE model a small and stable term premium whereas Hordahl *et al* (2007), Ravenna and Seppala (2006) find a sizeable and variable term premium. Rudebusch and Swanson (2008) attribute the result in Hordahl *et al* (2007), Ravenna and Seppala (2006) to large and persistent shocks. It is interesting to address the prices of risk in DSGE models. First, they represent the relative importance of shocks when the size of volatility is controlled. Second, to my knowledge we do not understand yet how the prices of risk involved in the shocks are related to structural parameters. The impact of monetary policy actions on the size of risk premia is also studied in this paper.

As an analytical solution is unavailable, the model is solved by perturbation method and estimate the shocks by Simulated Method of Moments (SMM). SMM is an attractive method to estimate nonlinear DSGE models because, like Maximum Likelihood (ML), it delivers consistent parameter estimates (see Lee and Ingram, 1991, and Du e and Singleton, 1993) but, in addition, it is generally robust to misspecification and the computation of the statistical objective function is quite cheap (see Ruge-Murcia, 2007 and 2010).

The results indicate that second-order approximate solution delivers a positive risk premia leading to an upward sloping average term structure. Results also show that: 1) increases in the inflation parameter of the Taylor rule (a more agressive monetary policy) lead to decreases in risk premia. Because *leaning against the wind* decreases inflation volatility and then leads to a decrease in inflation risk premium; 2) preferences shock and technology shock are more important than monetary policy shock in terms of contribution to the level of risk premium. But this is only because the calibrated monetary policy shock is very low compared to the two other shocks volatility. In fact, the contribution of each shock to the size of risk premium is a result of two effects: the size of the volatility of the shock and the price per unit of risk involved in each shock. The preferences shock volatility in the benchmark model is ten time larger than the monetary policy volatility that makes its combined effect larger than the other shocks contributions; 3) The price of risk associated with the monetary policy shock is larger than the other shocks price and is increasing with the maturity. Preference shock associated risk price is the lest important; 4) habit strength parameter has less impact on the risk premia when the capital stock is allowed to vary over time but the impact becomes important when the adjustment in capital is high enough to induce a fixed capital stock.

The rest of the paper is organized as follow. Section 2 presents the model, Section 3 is devoted to derive interest rates and risk premia from the equilibrium conditions as functions of macroeconomic factors. In Section 4 we present the solution method and calibrate the model in section 5. Finally, Section 6 presents the results and discusses some sensitivity analyses.

2 The Model

The model features a standard New-keynesian economy wherein a representative consumer derives utility from a composite consumption good and leisure. The composite good is produced by a representative firm with a continuum of intermediate inputs goods. Consumers can save resources by using nominal bonds or capital. There is a central banker who adjusts the nominal short-term interest rate according to a Taylor-type rule

2.1 Households

The representative consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t A_t \left(\frac{(c_t - bc_{t-1})^{1-\gamma}}{1-\gamma} - \psi \frac{n_t^{1+\phi}}{1+\phi} \right), \tag{1}$$

where E_0 is the mathetical expectation given the time 0 information set, $\beta \in (0,1)$ is the subjective discount factor, $b \in [0,1)$ is habit strength parameter, γ and ψ are constant preference parameters, ϕ is the inverse of Frisch elasticity of labor supply, A_t is a preference shock, c_t is a composite index of a continuum of intermediate goods, c_t^i, $i \in [0\ 1]$, n_t is hours worked. We assume internal habit in the composite consumption index c_t defined by:

$$c_t = \left[\int_0^1 (c_t^i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1$$

The parameter θ is the elasticity of substitution between the individual goods. As $\theta \to \infty$, the intermediate goods become closer substitutes and the weaker the firm's power on these goods.

Resources can be intertemporally transferred through assets including cash balances, capital and private nominal bonds with maturities $\ell = 1, \ldots, L$. The consumer budget constraint is

$$\int_0^1 \frac{p_t^i c_t^i}{P_t} di + x_t + \frac{M_t}{P_t} + \sum_{\ell=1}^{L} \frac{Q_t^\ell B_t^\ell}{P_t} = \frac{W_t n_t}{P_t} + \frac{R_t k_t}{P_t} + \frac{M_{t-1}}{P_t} + \sum_{\ell=1}^{L} \frac{Q_t^{\ell-1} B_{t-1}^\ell}{P_t} + \frac{S_t}{P_t} - \Gamma\left(\frac{x_t}{k_t}\right) k_t, \tag{2}$$

where k_t is capital, x_t is investment, $\delta \in (0,1)$ is the capital depreciation rate, Q_t^ℓ and B_t^ℓ are, respectively, the nominal price and holding of bond with maturity ℓ , W_t is the nominal wage, R_t is the nominal rental rate per unit of capital, p_t^i is the price of the intermediate good i, , M_t is nominal money balances, and P_t is the aggregate price level, S_t is a lump-sum tax or transfer. Note that an ℓ-period bond at time $t-1$ becomes an $(\ell-1)$-period bond at time t. Capital accumulation is subject to adjusment cost that is a function of the investment-capital stock ratio x_t/k_t. The law of motion of the capital accumulation is

$$k_{t+1} = (1-\delta)k_t + \Gamma\left(\frac{x_t}{k_t}\right) k_t, \tag{3}$$

where $\Gamma(\cdot)$ is a strictly convex function of x_t/k_t. For simplicity, we assume a quadratic function for Γ with no adjustment cost in the steady state. That is,

$$\Gamma\left(\frac{x_t}{k_t}\right) = \frac{\varphi}{2}\left(\frac{x_t}{k_t} - \delta\right)^2$$

In a first stage the consumer shops for intermediate goods for the composite good production. Given a level of the composite good, the consumer chooses the inputs c_t^i, $i \in [0\ 1]$ that minimize the total cost $\int_0^1 p_t^i c_t^i di$. This implies that demand for an intermediate good i is given by:

$$c_t^i = \left[\frac{p_t^i}{P_t}\right]^{-\theta} c_t,$$

and the aggregate price level P_t is given by:

$$P_t = \left[\int_0^1 (p_t^i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

The above expressions of demand functions for goods i and price index imply that:

$$P_t c_t = \int_0^1 p_t^i c_t^i di,$$

The budget constraint becomes:

$$c_t + x_t + \frac{M_t}{P_t} + \sum_{\ell=1}^{L} \frac{Q_t^\ell B_t^\ell}{P_t} = \frac{W_t n_t}{P_t} + \frac{R_t k_t}{P_t} + \frac{M_{t-1}}{P_t} + \sum_{\ell=1}^{L} \frac{Q_t^{\ell-1} B_{t-1}^\ell}{P_t} + \frac{S_t}{P_t} - \Gamma\left(\frac{x_t}{k_t}\right) k_t, \tag{4}$$

The household maximization problem is subject to (4) and (3)

The preference shock follows the process

$$\ln(A_t) = (1-\rho)\ln(A) + \rho \ln(A_{t-1}) + \sigma_u u_t, \tag{5}$$

where $\rho \in (-1, 1)$, $\ln(A)$ is the unconditional mean of $\ln(A_t)$, and u_t is assumed to be an independently and identically distributed (*i.i.d.*) innovation with mean zero and standard deviation equal to one. $\sigma_u > 0$ is constant parameter

The first-order conditions for the consumer's problem include Euler equations for capital, investment, and bonds:

$$1 = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[q_t r_{t+1} + \left(1 - \delta + \Gamma(\frac{x_{t+1}}{k_{t+1}}) + \frac{x_{t+1}}{k_{t+1}} \Gamma'(\frac{x_{t+1}}{k_{t+1}}) \right) \frac{q_t}{q_{t+1}} \right] \right\}, \tag{6}$$

$$q_t = 1 + \Gamma'\left(\frac{x_t}{k_t}\right), \tag{7}$$

$$Q_t^\ell = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{Q_{t+1}^{\ell-1}}{\pi_{t+1}} \right) \text{,for } \ell\text{=1,2,...L}, \tag{8}$$

where λ_t is the the marginal utility of consumption $r_{t+1} = 1 + \frac{R_t}{P_t} - \delta$ is the real return of capital, $\pi_{t+1} = P_{t+1}/P_t$ is the gross rate of inflation between time t and $t+1$, and q_t is the ratio of Lagrangian multipliers of constraint (4) and (3), that is the Tobin's q.

2.2 Firms

There are a final good competitive firm and a continuum of monopolistic firms that operate competitively.

2.2.1 Final Good Producer

The final good producer behaves in a perfectly competitive manner and takes as given the prices of intermediate goods and the aggregate price index when maximizing profits. The final good is produced using only the individual goods y_t^i as inputs in the following production function:

$$y_t = \left[\int_0^1 (y_t^i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}},$$

where y_t the quantity of the final good. Profit maximization implies that demand of input i is given by:

$$y_t^i = \left[\frac{p_t^i}{P_t} \right]^{-\theta} y_t, \tag{9}$$

2.2.2 Intermediate Goods Firms and Price Setting

Individual good $i \in (0\ 1)$ is produced by a monopolist through the following technology:

$$y_t^i = Z_t F(K_t^i, N_t^i), \tag{10}$$

where y_t^i is ouput, K_t^i is firm i capital demand, N_t^i is labor input and the function $F(.,.)$ is constant return to scale, strictly increasing and strictly concave in both of its arguments and satisfy the Inada conditions, Z_t is a total factor productivity shock that affects all firms in the same way.

The technology shock follows the process:

$$\ln(Z_t) = (1-\omega)\ln(Z) + \omega \ln(Z_{t-1}) + \sigma_\varepsilon^2 \varepsilon_t, \tag{11}$$

where $\omega \in (-1, 1)$, $\ln(Z)$ is the unconditional mean of $\ln(Z_t)$, and ε_t is a disturbance term assumed to be an independently and identically distributed (*i.i.d.*) with mean zero and standard deviation equal to one.

Intermediate good producing firm $i \in (0\ 1)$ hires labor and capital in perfectly competitive makets to produce its good. Firms are owned by households who receive any profit made by firms at each period.

Prices are set following the mechanism described in Calvo (1983) : each period a fraction of $1-\theta_p$ randomly picked firms can reset their price while the remaining fraction θ_p cannot. Those who have the opportunity to adjust their price, set it optimally to maximize their discounted profit while those who cannot adjust optimally, just set their price to the previous aggregate price level indexed by the steady state inflation. Note that θ_p governs the prices stickiness. The smaller θ_p is, the more flexible prices will be as firms will get to reset their price frequently.

The firm $i's$ problem is to choose K_t^i, N_t^i, p_t^i to maximize discounted profit subject to its good demand function, the production technology (10) and the price setting scheme. This can be done in two steps: first choose the capital and labor input to minimize the real cost given the production function (10) and given the real wage and capital rental rates. Second choose the price to maximize the discounted real profit subject to the demand function and given the aggregate price and quantities.

The real cost minization program is:

$$\underset{K_t^i,\ N_t^i}{Min} \left[w_t N_t^i + r_t K_t^i\right]$$
s.t $y_t^i = Z_t F(K_t^i, N_t^i) = Z_t (K_t^i)^\alpha (N_t^i)^{1-\alpha}$

The first order conditions imply that:

$$\frac{K_t^i}{N_t^i} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t} \tag{12}$$

Thus, all firms will choose the same capital-labor ratio. Using this relation, the real cost is given by:

$$Cost_t = w_t N_t^i + r_t K_t^i = \frac{1}{1-\alpha} w_t N_t^i$$

Use the production function and (12) to express N_t^i as a function of y_t^i, w_t, and r_t and substitute into the cost function to get:

$$Cost_t = \frac{y_t^i}{Z_t}\left[\frac{w_t}{1-\alpha}\right]^{1-\alpha}\left[\frac{r_t}{\alpha}\right]^\alpha$$

The real marginal cost mc_t is equal to the derivative of the real cost with respect to y_t^i and is given as:

$$mc_t = \frac{1}{Z_t}\left[\frac{w_t}{1-\alpha}\right]^{1-\alpha}\left[\frac{r_t}{\alpha}\right]^\alpha \tag{13}$$

Note that the real marginal is independent of i meaning that all firms incur the same marginal cost.

Now in the second step, firms pick their price p_t^i to maximize:

$$E_t \sum_{s=\tau}^{\infty} (\theta_p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} \left[\frac{p_t^i}{P_{t+s}} - mc_{t+s} \right] y_{t+s}^i$$

subject to:

$$y_{t+s}^i = \left[\frac{p_t^i}{P_{t+s}} \right]^{-\theta} y_{t+s}$$

Notice that when maximizing the profit, firms take into account the fact that a price set at time t will remain the same with probability $(\theta_p)^s$ at time $t+s$. It means that when θ_p is large, a price set in the current period will likely remain for a long period of time. Thus when choosing current price, firms will relatively weight more future profits.

Replace the demand function in the objective and take the derivative with respect to p_t^i gives:

$$E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} \left[(1-\theta) \left[\frac{p_t^i}{P_{t+s}} \right]^{1-\theta} \frac{1}{p_t^i} - \theta \left[\frac{p_t^i}{P_{t+s}} \right]^{-\theta} \frac{1}{p_t^i} mc_{t+s} \right] y_{t+s} = 0$$

Solve this equation to get

$$p_t^i = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} P_{t+s}^{\theta} mc_{t+s} y_{t+s}}{E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} P_{t+s}^{\theta-1} y_{t+s}} \tag{14}$$

The above equation (14) says that when firms have the opportunity to adjust their price, they optimally set it as some weighted mean of expected future nominal marginal costs.

The infinite summations implied in (14) make the computation tricky because we do not have a direct recursive formulation of the this expression. To get around this problem, let define the following auxilliary variables:

$$V_t = E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} P_{t+s}^{\theta} mc_{t+s} y_{t+s}$$

$$J_t = E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} P_{t+s}^{\theta-1} y_{t+s}$$

Then (14) becomes

$$p_t^i = \frac{\theta}{\theta - 1}\frac{V_t}{J_t}$$

Where the infinite sums V_t and J_t have the following recursive forms:

$$V_t = \theta_p \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} V_{t+1}\right] + P_t^{\theta} mc_t y_t \tag{15}$$

$$J_t = \theta_p \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} J_{t+1}\right] + P_t^{\theta-1} y_t \tag{16}$$

When all firms are able to adjust their prices each period ($\theta_p = 0$), price are set to markup ($\mu = \frac{\theta}{\theta-1}$) over nominal marginal cost ($P_t mc_t$)

$$p_t^i = \frac{\theta}{\theta - 1} P_t mc_t$$

whereas when $\theta_p > 0$, the optimal price is set as a markup over expected future weighted marginal costs. Notice that in the flexible price framework, the marginal is constant and equal to the inverse of the markup.

Because all firms face the same demand function, they will choose the same price when reoptimizing at time t, that is $p_t^i = p_t^j = p_t^*$ for those who are able the adjust and $p_t^i = P_{t-1}$ for those who cannot. Thus the price index is given by:

$P_t = \left[\theta_p P_{t-1}^{1-\theta} + (1-\theta_p) P_t^{*1-\theta}\right]^{\frac{1}{1-\theta}}$ and the inflation rate

$$\frac{P_t}{P_{t-1}} = \left[\theta_p + (1-\theta_p)\frac{P_t^{*1-\theta}}{P_{t-1}^{1-\theta}}\right]^{\frac{1}{1-\theta}}$$

where $\frac{P_t^{*1-\theta}}{P_{t-1}^{1-\theta}} = \left[\frac{\theta}{\theta-1}(1+\pi_t)\frac{\widetilde{V}_t}{\widetilde{J}_t}\right]^{1-\theta}$ and $\widetilde{V}_t = \frac{V_t}{P_t^{\theta}}$, $\widetilde{J}_t = \frac{J_t}{P_t^{\theta-1}}$

Inflation can then be solve out as a function of $\widetilde{V}_t$ and $\widetilde{J}_t$ from this equation

$$1 + \pi_t = \left[\theta_p + (1-\theta_p)\left[\frac{\theta}{\theta-1}(1+\pi_t)\frac{\widetilde{V}_t}{\widetilde{J}_t}\right]^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{17}$$

Where from (15) and (16), $\widetilde{V}_t$ and $\widetilde{J}_t$ evolve according to:

$$\widetilde{V}_t = \theta_p \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\theta} \widetilde{V}_{t+1}\right] + mc_t y_t \tag{18}$$

$$\widetilde{J}_t = \theta_p \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\theta-1} \widetilde{J}_{t+1}\right] + y_t \tag{19}$$

The production side equilibrium conditions are given by (9) - (13), (17) - (19).

2.3 Monetary Policy Rule and Government

Fiscal policy consists of lump-sum transfers to households each period which are financed by newly printed money. The government budget constraint is given by:

$$\frac{S_t}{P_t} = \frac{M_t - M_{t-1}}{P_t}, \tag{20}$$

The model is closed with a Talor-type policy rule whereby the monetary authority sets the one-period nominal interest rate as a function of inflation and ouput deviations from targetted levels.

$$\frac{1+i_{t+1}}{1+i^{ss}} = \left(\frac{1+i_t}{1+i^{ss}}\right)^{\rho_i} \left(\frac{1+\pi_t}{1+\pi^{ss}}\right)^{(1-\rho_i)\gamma_\pi} \left(\frac{y_t}{y^{ss}}\right)^{(1-\rho_i)\gamma_\pi} \exp(mp_t) \tag{21}$$

where i_t is the time t one-period nominal bond ineterest rate, mp_t is monetary innovation and i^{ss}, π^{ss}, y^{ss} are steady values of the short term nominal interest rate, inflation and output.

2.4 Market Clearing and Aggregation

Using the fact that the capital-labor ratio is firm independent, we can get the aggregate capital-labor ratio

$$\frac{K_t}{N_t} = \int_0^1 \frac{K_t^i}{N_t^i} di = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}$$

The aggregate supply over firms is then given by:
$\int_0^1 y_t^i di = Z_t \left(\frac{K_t}{N_t}\right)^\alpha \int_0^1 N_t^i di$ and the aggregate demand is $y_t \int_0^1 \left(\frac{p_t^i}{P_t}\right)^{-\theta} di.$

In equilibrium aggregate demand must equal the aggregate supply or

$$y_t \int_0^1 \left(\frac{p_t^i}{P_t}\right)^{-\theta} di = Z_t K_t^\alpha N_t^{1-\alpha} \tag{22}$$

where $N_t = \int_0^1 N_t^i di$.

From (22), the aggregate composite index of ouput is

$$y_t = \frac{Z_t K_t^\alpha N_t^{1-\alpha}}{d_t} \tag{23}$$

where $d_t = \int_0^1 \left(\frac{p_t^i}{P_t}\right)^{-\theta} di$ is the price dispersion and introduces a distorsion in output aggregation. The fact that firms choose different prices in equilibrium can lead to aggregate output lost when. In fact, firms who choose to increase their relative price will face a decrease in the demand of their good and then a decrease in their output.

When prices are flexible, all firms choose the same price and their is no distorsion in aggregate output, that is, d_t is always equal to 1.

The Calvo pricing structure implies that the law of motion of d_t is given by:

$$d_t = \theta_p \left[\frac{1}{1+\pi_t}\right]^{-\theta} d_{t-1} + (1-\theta_p)\left[\frac{\theta}{\theta-1}\frac{\widetilde{V}_t}{\widetilde{J}_t}\right]^{-\theta} \tag{24}$$

In equilibrium all the markets must clear every period:

$c_t + x_t = y_t$

$k_{t+1} = (1-\delta)k_t + x_t - \Gamma\left(\frac{x_t}{k_t}\right)k_t$

$n_t = N_t = \int_0^1 N_t^i di$

$k_t = K_t = \int_0^1 K_t^i di$

$B_t^\ell = 0$ for all $\ell = 1, 2, ..., L$

2.5 Equilibrium

Definition: An equilibrium is an allocation for the household $\mathcal{C} = \{c_t, n_t, x_t, k_{t+1}\}_{t=0}^{\infty}$, $\left\{(B_t^\ell)_{\ell=1,..L}\right\}_{t=0}^{\infty}$, an allocation for the firm $\mathcal{F} = \{Y_t, K_t, N_t\}_{t=0}^{\infty}$, a prices system $\{\pi_t, W_t/P_t, R_t/P_t\}_{t=0}^{\infty}$, $\left\{(Q_t^\ell)_{\ell=1,..L}\right\}_{t=0}^{\infty}$ such that given k_0 and the prices system:

1) the allocations $\mathcal{C}$ and $\mathcal{F}$ solve the households' and the firms' problems,

2) good market clears: $Y_t = C_t + x_t + \Gamma\left(\frac{x_t}{k_t}\right)k_t$,

3) $n_t = N_t = 1$,

4) bonds are in zero net supply: $B_t^\ell = 0$ for all $\ell = 1, 2, ..., L$.¶

5) The money market clears.

In the following section, we review the relation between the bond prices implied by the economic model and the term structure of interest rates, and define risk premia. From the bond prices implied by the first-order conditions of bonds demand, we derived the term structure of interest rates and expressions for risk premia as functions of macroeconomic fundamentals.

3 Interest Rates and Risk Premia in DSGE Models

New Keynesian DSGE models are well known to be able to reproduce salient features of macroeconomic data (see, Smets and Wouters, 2004) but fail to match simultaneously financial and macro data. In fact, matching risk premia involved in financial assets is a challenging issue for DSGE modellers, yet it is easy to reproduce risk premia in an exchange economy by adding some real frictions such as habit formation in a standard RBC model (see Wachter, 2006 and Piazzesi and Schneider, 2006) . With habit formation preferences, current consumption levels affect future marginal utilities and the risk aversion is countercyclical instead of being constant as in RBC models. This allows the model to calibrate high steady state risk aversion with a reasonnable consumption curvature parameter (see, Campbell and Cochrane, 1999), and then to generate sizeable risk premia consistent with the data. In a production economy where consumption, hours worked and ouput are endogenous, there are available channels to the consumer for overcoming bad income shocks, that are absent in exchange economies. Thus in terms of consumption smoothing, risk averse consumers claim bigger premium to hold a long-term bond in exchange economies because they are more exposed to income uncertainty in endowment economy than in production economy. For example, consumers will be able to work more in production economies to increase their income when they face bad income shocks; whereas this chanel is absent in exchange economies. As a consequence, the increasing effect of habit formation on risk premia is weakened in a production economy wherein consumers can adjust labor or accumulate capital.

Define the gross interest rate of the one-period bond as

$$i_{1,t} = \frac{1}{Q_t^1} \tag{25}$$

More generally, the gross nominal interest rate of the ℓ-period bond is defined as

$$i_{\ell,t} = \left[Q_t^\ell\right]^{\frac{-1}{\ell}} \tag{26}$$

There are various formulas of risk premiums in the literature but Rudebusch *et al.* (2007) show that they are highly correlated. The overall risk involved in long-term nominal bonds is twofold: first, there is a risk of capital loss in the future in case of resaling the bond before the maturity date. Because the bond future prices are not known with certainty in advance, the eventual resale[4] price could be less than the purchase price. Second, there is an inflation risk involved in nominal long-term bonds because inflation can erode the bond value in the future. The risk premium can be derived recursively from Euler equation for bonds,

[4] For example in case of a negative realization of an income shock somewhere between t and $t+\ell$, an $\ell-$period bond holder would like to redeem the bond in order to smooth its consumption

$$Q_t^{\ell} = Q_t^1 E_t\left(Q_{t+1}^{\ell-1}\right) + \beta cov_t\left(Q_{t+1}^{\ell-1}, \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}}\right), \tag{27}$$

where we have used the fact that the one-period bond price is

$$Q_t^1 = E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}}\right). \tag{28}$$

The $\ell-$period term-premium, denoted by $TP_{\ell,t}$, is usually defined as the difference between an $\ell-$period interest rate and expected average of short-term rates over the maturity period, that is,

$$TP_{\ell,t} = i_{\ell,t} - \frac{1}{\ell}E_t\sum_{s=0}^{\ell-1} i_{1,t+s} \tag{29}$$

A similar form of (29) in our model is captured by the covariance term of the right hand side of (27). The risk premium we will use in this model, is the excess holding period return, that is, the return from holding an $\ell-$period bond for one period relative to the return of one-period bond[5]. We can rearrange (27) to get

$$E_t\left[\frac{Q_{t+1}^{\ell-1}}{Q_t^{\ell}}\right] = \frac{1}{Q_t^1} - cov_t\left[\frac{Q_{t+1}^{\ell-1}}{Q_t^{\ell}}, \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{1+\pi_{t+1}}\frac{1}{Q_t^1}\right] \tag{30}$$

At time t+1, an $\ell-$period bond will become an $(\ell-1)$ maturity bond such that the gross holding period return $H_{\ell,t+1}$ is given by

$$H_{\ell,t+1} = \frac{Q_{t+1}^{\ell-1}}{Q_t^{\ell}}$$

From (30) we have,

$$E_t(H_{\ell,t+1}) = i_{1,t} + rp_{\ell,t} \tag{31}$$

where $rp_{\ell,t} = -cov_t\left[H_{\ell,t+1}, \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{1+\pi_{t+1}}i_{1,t}\right]$ is the holding period risk-premium.

It is easy to show that

$$TP_{\ell,t} = \frac{1}{\ell}E_t\sum_{s=0}^{\ell-1} rp_{\ell-s,t+s}$$

The term-premium is thus the mean of all expected holding period risk-premia

[5] Computationally, the excess holding period return requires less complementary state variables definition than the term premium

(31) says that after ajusted for risk factor, the holding-period return is a predictor of the one-period interest rate. Note that this covariance term can either be positive or negative depending on the direction of the covariation between the holding-period return and the nominal discount factor. When high future marginal utility- that is the situation where the investor needs more consumption- is associated with capital losses ($Q_{t+1}^{\ell-1}$ is low relative to Q_t^{ℓ} when reselling an $\ell-$period bond at t+1), investors will claim a positive risk premium for holding a long term bond instead of short-term bonds. We can also notice the two sources of risk involved in long-term nominal bonds highlighted above. First, the term premium is affected by the covariation between the holding-period return and the real stochastic discount factor keeping the inflation rate constant. Second, correlation between the holding-period return and future inflation rate, keeping the real stochastic discount factor constant, also determines the sign and the size of the risk premia. In the first case, the resulting term premium will be referred as the real term premium and in the second case the inflation term premium. The sign and the magnitude of the total term premium will depend on the combination of these two covariance effects.

In general inflation risk premium compensates the bond holder for the inflation risk involved in keeping a nominal asset rather than a riskless real asset. In our model, such an asset could be the capital stock would the productivity shock be constant over time and without adjustment cost in capital. Use the first-order conditions for the one-period bond to get

$$Q_t^1 = E_t(\beta \frac{\lambda_{t+1}}{\lambda_t}) E_t(1/(1+\pi_{t+1})) + cov_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t}, \frac{1}{1+\pi_{t+1}} \right],$$

where the covariance term is the one-period inflation risk premium denoted by $Inflpr_t^1$ because this conditional covariance is 0 when the inflation process is deterministic. Similarly the $\ell-$period inflation risk premium is defined from the $\ell-$period bond Euler equation as:

$$Inflpr_t^{\ell} = Q_t^{\ell} - E_t(\beta^{\ell} \frac{\lambda_{t+\ell}}{\lambda_t}) E_t(1/(1+\pi_{t+\ell})) = cov_t \left[\beta^{\ell} \frac{\lambda_{t+\ell}}{\lambda_t}, \frac{1}{1+\pi_{t+\ell}} \right]$$

where $\pi_{t+\ell} = \frac{P_{t+\ell}}{P_t}$

4 Model Solution

Since the model does not have an exact analytical solution, we use perturbation method that involves taking a second-order expansion of the policy rules around the deterministic steady state. For detailed explanations of this approach, see Jin and Judd(2002), Schmitt-Grohé and Uribe(2004), and Kim, Kim, Schaumburg and Sims(2008). Perturbation methods deliver a zero risk-premium at first-order approximation due to the certainty equivalence at first-order and a constant risk-premium at second-order approximation.

The standard approach of perturbation method writes the model general equilibrium conditions in the form:

$$E_t F(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \tag{32}$$

where E_t is the conditional expectation given the time t information set, y_t is the vector of controle variables and x_t the predetermined endogenous variables and exogenous processes. F is a vectoral function of all the equilibrium conditions. The solution of the model is given by:

$$y_t = g(x_t, \sigma)$$

$$x_{t+1} = h(x_t, \sigma) + \sigma\eta\varepsilon_{t+1}$$

where h and g are unknown functions, η is constant matrix driving the variances of the innovations and σ is a scaling perturbation parameter driving the size of the uncertainty in the economy. Given that h and g are unknown, the procedure consists of approximating the functions h, g around the non-stochastic steady state point $(x, 0)$ where uncertainty is removed. The approximate solution takes the form:

$$y_t = y + \frac{1}{2} g_{\sigma\sigma}\sigma^2 + g_x(x_t - x) + \frac{1}{2}(I_{ny} \otimes (x_t - x))' g_{xx}(x_t - x) \tag{33}$$

$$x_{t+1} = x + \frac{1}{2} h_{\sigma\sigma}\sigma^2 + h_x(x_t - x) + \frac{1}{2}(I_{nx} \otimes (x_t - x))' h_{xx}(x_t - x) + \sigma\eta\varepsilon_{t+1} \tag{34}$$

where $x = h(x, 0)$ and $y = g(x, 0) = g(h(x, 0), 0)$ and n_y and n_x are the number of controle and state variables respectively, I is an identity matrix. g_x, h_x, g_{xx},

h_{xx}, $h_{\sigma\sigma}$ are constant coefficients standing for first and second derivatives of g and h with respect to x and σ evaluated at the deterministic steady state. Notice that these coefficients are functions of the structural parameters of the model and that the parameter σ enters the decision rules as an argument capturing the risk factors.

The constant risk premia delivered by the second-order approximate solution is a combination of volatilities of the shocks. Thus to understand the determinants of risk premia in DSGE models, it is useful to write the second-order risk-premium as

$$rp_\ell = \frac{1}{2} rp_\ell^a \sigma_a^2 + \frac{1}{2} rp_\ell^z \sigma_z^2 + \frac{1}{2} rp_\ell^{mp} \sigma_{mp}^2 \tag{35}$$

where rp_ℓ^a, rp_ℓ^z, rp_ℓ^{mp} are functions of structural parameters and σ_a^2, σ_z^2, σ_{mp}^2, are the volatility of preference, productivity and monetary policy shocks respectively.

5 Estimation

5.1 Data

I estimate the model using U.S. macroeconomic as well as term structure data at the quarterly frequency. The sample period is 1962 Q1 -2001 Q2 and is limited by the availability of the term structure data.

The macro data used are real consumption growth, real GDP growth, and Consumer Price Index (CPI) inflation rate. Consumption is NIPA measures of personal consumption expenditure on non durable goods and services. Real consumption is obtained by dividing it nominal measure by CPI inflation rate. All the macro data are taken from the Federal Reserve Bank of St. Louis website (*www.stls.frb.org*) and are seasonally adjusted at the source.

The term structure of interest rates data are the nominal three-month interest rate and the ten-year nominal interest rate. The three-month rate is treasury bill whereas the ten-year interest rate is constant maturity rate. All interest rates are also taken from the Federal Reserve Bank of St. Louis website. The original interest rates were available at a daily frequency. Quarterly observations have been obtained by taking the first trading day observation of the second month of each quarter[6] (February, May, August, November). These interest rates are well suited to the theoretical interest rates of the model because first, they are interest rates on zero-coupon bonds and second the default risk is negligible.In the estimation, I use the spread between the ten-year and the three-month nominal interest rates. Since a period corresponds to a

[6] Instead of averaging over the quarter

quarter in the model, the empirical counterparts of the one-period and forty-period interest rates are respectively three-month and ten-year interest rates.

5.2 Paramaters Estimation: Simulated Method of Moments (SMM)

I estimate the shocks parameters of the model by the Simulated Method of Moments (SMM). The number of estimated parameters is five : the persistence parameters of technology (ρ_a) and preferences (ρ_u) shocks; and the standard deviations of the three shocks σ_u, σ_ε, σ_{mp}. The remaining parameters have been calibrated to the U.S. economy or set in line with the literature.

SMM consists in minimizing a weighting distance between unconditional moments predicted by the model and the corresponding data moments counterpart. Basically, the predicted moments are based on artificial data simulated from the model while data moments are directly computed from actual data. SMM is an attractive method to estimate nonlinear DSGE models because, it delivers consistent parameter estimates (see, Lee and Ingram (1991), Duffie and Singleton (1993)). Moreover, as shown in Ruge-Murcia (2007), SMM is generally robust to misspecification and the computation of the statistical objective function is quite cheap. Ruge-Murcia (2010) explains in detail the application of SMM for the estimation of higher-order DSGE models and provides Monte-Carlo evidence on its small-sample properties. Since Ruge-Murcia (2010) shows that SMM based asymptotic standard errors tend to overstate the actual variability of the estimates, I use a block bootstrap method to compute a ninety- five per cent confidence intervals for the parameter estimates.

I simulate the model on the basis of the prunned version of the second-order approximate solution proposed by Kim, Kim, Schaumburg and Sims (2008). The innovations are drawn from the normal distribution for the simulation. The moments used in the estimation are the variances, first- and second-order autocovariances of the four data series, in addition to the unconditional mean of the interest rate spreads and the inflation rate. Because consumption growth and real GDP growth rates are positive in the U.S. data and there is no growth in the model, I discard the mean of these two variables. Thus, forteen moments are used in the estimation of the five parameters meaning that the number of degrees of freedom is nine. The weighting matrix used is the diagonal of Newey-West estimator of long-run variances of the moments with a Bartlett kernel and bandwidth given by the integer of $4(\mathrm{T}/100)^{2/9}$ where T is the sample size. The sample size here is T=158 which implied a bandwith value of 4.427. The number of the simulated observations is five times larger than the sample size T.

Because the theoretical properties of SMM estimates are valid under stationnarity assumptions, a unit root test has been performed on the series used in the estimation. To this end, I use an Augmented-Dickey Fuller (ADF) and a Phillips-Perron (PP) unit root tests. The null hypothesis of unit root can be rejected at 5% level under both tests for all series except the inflation rate. However, for the inflation rate, the unit root hypothesis can be rejected at the 5% level under the PP test but cannot be rejected under the ADF test. But the ADF-statistic is -2.38 whereas the critical value is -2.39. So, I suppose that the inflation rate is stationnary.

5.3 Calibration

During the estimation, the remaining model parameters have been calibrated as follow:

The subjective discount factor is parametrized at $\beta = 0.99$ to match the average annual real interest rate of 4%. The consumption curvature coefficient in the utility function is set to a value of $\gamma = 2$. This value is in the range of empirical estimates in the DSGE models literature [7]. The habit strength parameter is set to $b = 0.65$ as in Constantinides (1990), Boldrin, Christiano and Fisher (2001). The labor elasticity is set to $\varphi = 2$ and φ_0 is calibrated to match 1/3 of steady state hours worked without habit in consumption[8] as found in the U.S. post war II data. The depreciation rate of capital is set to 0.025 per quarter such that the steady state investment-output ratio is 23%. The capital adjustment cost parameter is set to $\phi = 10$.

In the production side, the most important parameters that need to be discussed are the Calvo parameter θ_p which controls the price stickiness and the parameter θ representing the firms power. The model steady state mark-up Ψ is given by the expression $\Psi = \frac{\theta}{\theta-1}$. In the data, the long-run mark-up has been estimated to be about 10% meaning that $\frac{\theta}{\theta-1} = 1.1$. This implies a value of $\theta = 11$.

The Calvo parameter or the proportion of resetting price firms θ_p is also related to the average duration of a price set at time t. Conditional on setting optimally a price at time t, the probability of being able to reset optimally for the first time at time $t+j$ is $(1-\theta_p)\theta_p^j$. It means that the average duration of a price set at time t is: $D = \sum_{j=0}^{\infty} j(1-\theta_p)\theta_p^j = \frac{1}{1-\theta_p}$ which implies an average duration of price changes of

[7] For example Smets and Wouters (2005) find an estimate of $\gamma = 2.6$

[8] with habit formation preferences the implied steady state of labor is about 1/2

1 year. Thus θ_p can be calibrated by computing the average duration between price changes. The range of estimates by Bils and Klenow (2004) of average price changes in micro data is between six months and one year. Setting $D = 1$ year, that is 4 quarters yields $\frac{1}{1-\theta_p} = 4$ or $\theta_p = 0.75$.

I compute the U.S. long run money gross growth rate (1.01) and set the steady state gross inflation rate to this value. With the annual 4% steady state real interest rate, this implies an annual steady state of nominal interest rate of 8%. The non-zero steady state inflation rate implies a steady state of real marginal cost (0.87) that is sligthly different from the inverse of the mark-up (0.91). The production function parameter α is set at 0.41 to match a long run U.S. capital share of income of 0.37.

There are no standard values of the monetary policy parameters and different papers (Clarida, Gali and Gertler (1999) among others) have estimated different values. In the New Keynesian literature, these parameters are usually chosen in range which satisfies the equilibrium stability. I follow Ravenna and Seppala (2005) and Rabanal and Rubio-Ramirez (2004) to choose these parameters. The inflation reaction parameter and the output reaction parameter are respectively set to $\gamma_\pi = 3$, $\gamma_y = 0.1$.

Table 1: baseline calibrated parameters

parameters	**description**	**value**
β	subjective discount factor	0.99
γ	consumption curvature	2
ϕ	adjustment cost parameter	10
α	share of capital income	0.41
δ	depreciation rate	0.025
θ	elasticity of substitution among goods	11
θ_p	proportion of firms not adjusting price	0.75
π^{ss}	long-run gross inflation rate	1.01
γ_π	inflation coefficient in Taylor rule	3
γ_y	output coefficient on Taylor rule	0.01

5.4 Parameters Estimates

Table 2 reports the SMM estimates of the shocks parameters. The Productivity shock is very persistent and volatile. The estimates of the autocorrelation coefficient (ρ_a =0.981) and standard deviation (σ_a =0.0105) are relatively well precise. The preferences shock is mildly persistent (ρ_u =0.553) but very volatile (σ_u =0.047). The monetary policy shock has been contrained to an *i.i.d.* process and the estimated volatility is very low (σ_{mp} =0.95x10^{-4}). This means that the dynamics of the model is mostly driven by productivity and preferences shocks.

Table 2: SMM Estimation

Description	Symbol	Estimates
Persistence parameter of productivity shock	ρ_a	0.981 (0.955, 0.985)
Persistence parameter of preferences shock	ρ_u	0.553 (0.303, 0.672)
Standard deviation of productivity shock	σ_a	0.011 (0.007, 0.013)
Standard deviation of preferences shock	σ_u	0.047 (0.039, 0.061)
Standard deviation of monetary policy shock	$\sigma_m p$	0.95×10^{-4} $(0.83 \times 10^{-4}, 0.27 \times 10^{-3})$

Note: block bootstrap 95% confidence intervals in parenthesis

Table 3: Unit Roots Test

	Test Statistic	
Variable	ADF	PP
Growth Rate of GDP	-6.435*	-9.242^{*}
Growth Rate of Consumption	-4.369*	-7.95^{*}
Rate of Inflation	-2.102	-3.148^{**}
Interest Rates Spread 10 year - 3 month	-3.854^{*}	-4.274^{*}

Note: **,* indicate significance at the 1%, 5% levels, respecively.

6 Results and Sensitivity Analysis

The results of the calibrated second-order approximate solution of the model and sensitivity exercises are presented in this section.

6.1 The term structure of interest rates

I present in this part the model implied term structure statistics. Note that with the calibrated 1% of quartely long-run inflation rate, the steady state of nominal interest rate of 8% (annual). I simulate the model and report the unconditional mean of interest rates statistics in Table 2. As Figure 1 shows, the model generates an upward sloping average term structure of interest rates. The model is able to generate a positive risk premium which leads to the upward sloping term structure. Table 2 shows that the risk premium is increasing in maturity. For example, the 4-period term premium is 0.5 basis points and the 10-period term premium is 1.8 basis points (annualized). Notice that the empirical counterpart of the 10-period maturity is 2.5 years as the model is calibrated to a quarterly basis.

Table 4: Model implied term structure statistics, baseline calibrated parameters

	Value(%)
TP_2	0.1
TP_3	0.2
TP_4	0.5
TP_{10}	1.8
r	4
I1	8
I2	8.5
I3	8.57
I4	8.87
I10	10

Now, I study the impact of monetary policy shock, preferences shock and productivity shock on selected variables of the model. Because the model is non-linear (2nd-order approximation), I present variables responses to positive as well as negative shock to capture potential asymmetric responses. The blue line is response to positive shocks and the green line response to negative shocks.

Figure 2 presents the impulses responses of key variables to a 1 standard deviation of monetary policy shock. As the standard deviation of monetary policy is 0.003, this corresponds to an annual 120 basis points (40000x1x0.003) increase in the short-term nominal interest rate. Figure 2 shows that an unexpected increase (decrease) in monetary policy shock leads to a decrease (increase) in consumption and inflation as expected. An increase (decrease) in monetary policy rate leads to an increase(decrease) in nominal bond interest rates. But the response decreases with the maturity such that shorter term nominal rates respond more than longer maturity rates. As a result, an increase in monetary policy rate will induce a decrease in interest rate spreads. Results in figure 2 also show that there is no asymmetric response for relatively small shocks.

Responses to productivity shock are reported in figure 3. The size of the shock is 1 standard deviation. A positive (negative) productivity shock leads to a decrease (increase) in nominal bond yields at all maturities. At the impact time, the magnitude of the response is the same across maturities. However, the persistence of the effect is increasing across maturities with the response of longer maturity interest rates more persistent than shorter maturity rates. It means that nominal spreads do not respond to productivity shock at the impact time but increase (decrease) after a positive (negative) productivity shock.

Suppose now the impact of preferences shock on the nominal interest rates. The results are reported in figure 4. This shock affects directly marginal utilities and the pricing kernel. A 1 standard deviation increase (decrease) in preferences shocks leads to an increase (decrease) in nominal bond interest rates. As in the case of monetary policy shock, the effect is decreasing across maturities with shorter term rates respond more than longer maturity rates.

6.2 Shocks contribution to risk permium

In this part I analyze the relative importances of each shock to the determination of the size of the risk premium. Remember that the second-order approximation implied risk premium is a weighted sum of the volatilities of the shocks. Thus as expression (35) shows, each shock contributes to the size of risk premia in two ways. First, the importance of each shock in terms of contribution will depend on its weighting coefficient. These coefficients can be interpreted as unitary prices of risk associated with each shock. That is, they capture the intensity of the agent's risk aversion towards the corresponding shocks. Second, shock volatility sizes are important to the determination of risk premia. The volatity sizes capture the quantity of risk involved in each shock and risk premia are expected to increase as the quantity of risk increases. The total contribution of a shock is then its unitary price of risk times the quantity of risk embedded in this shock.

This decomposition of shock contribution to risk premia is important because in the literature, the ability of a DSGE model to match risk premia statistics usually heavily relies on the calibrated (or estimated) shock size. The calibration of the shocks volatilities is sometimes controversial. For example, Hordahl, Tristani, and Vestin (2007) use a calibrated DSGE model with 2.3% standard deviation of technology shock. This allows their model to generate term premia large enough to be consistent with the data. This value of technology shock standard deviation is more than two times the standard value of 1% used in macro models. This is also the case in Ravenna and Seppala (2006) where the standard deviation of preferences shock has been set to 8% that is also large compared to the estimated value of 4% in Bansal *et all* (2005). Rudebusch and Swanson (2008) criticize the reliance of these two authors results to such large shock sizes. Furtheremore Ravenna and Seppala (2006) find that in New Keynesian framework, rejections of expectation hypothesis are explained by the systematic part of the monetary policy rather than by monetary innovations as found in Buraschi and Jiltsov (2006). However, the comparison of the two results is not clear since the two papers differ in many aspects including the nature and size of the shocks. Thus, for given shock sizes, we can compare different models ability to generate risk premia based on the second terms which capture the market prices of risk.

Table 3 shows the total contribution of each shock to the size of the baseline model risk premia. Preferences shock is far the most important shock in terms of contribution to the size of the risk premia and the contribution is increasing with the maturity. This result is not surprising because the calibrated preferences shock standard deviation is very large (0.04) relative to the two other shocks (more than 10 times) meaning preferences shock carries the largest quantity of risk to the point of view of the investor. As a consequence, the combined quantity and price of its associated risk gives the most important value relative to the two other shocks.

Table 5: Shock contribution to risk premia (baseline parameters

Shocks	rp_4	rp_8	rp_{10}	rp_{13}
Preference	69%	80%	81.5%	83%
Productivity	15%	13%	12.86%	12.7%
Monetary Policy	16%	7%	5.7%	4.3%

But instead of looking at the quantity effect one can also analyze the price effect of the risk carried by each shock. That will capture the relative undesirability of each shock given the same quantity of risk. To address this issue, I now analyze the model implied coeffeicients rp_ℓ^a, rp_ℓ^z, rp_ℓ^{mp} in equation (35).

Table 4 shows the model implied rp_ℓ^a, rp_ℓ^z, rp_ℓ^{mp}. It turns out that in terms of the price per unit of risk, monetary policy has the largest effect on the size of risk premia meaning that if all shocks were calibrated to have the same variance, monetary policy would have the most important contribution to the risk premia with more than 80% for the 4-period bond. This result can be interpreted as follow. Suppose an hypothetical economy where the three shocks have the same standard deviations. Then, monetary policy innovations are more undesirable *vis a vis* the investor relative to productivity and preferences shocks. Note also that all prices of risk are increasing with the maturity

Table 6: Prices of risk (baseline parameters)

$\ell =$	4	8	10
rp_{ℓ}^{a}	0.05	0.16	0.2
rp_{ℓ}^{z}	0.4	0.82	1.05
rp_{ℓ}^{mp}	2.3	2.5	2.57

6.3 Sensitivty Analysis

In this section, I do some sensitivity exercises by varying key structural parameters in order to understand the determinants of the size of the risk premia. For example, we want to know how the monetary policy actions part affect risk premia.

Monetary policy and risk premia: do monetary policy actions matter for risk premia? That is, how changes in monetary policy parameters affect risk premia. Results indicate that a more agressive monetary policy stance, that is, an increase in the inflation reaction parameter, leads to decreases in risk premia. This is because when the central bank leans against the wind, the inflation volatility decreases. Less volatile inflation means less uncertainty in inflation and then less inflation risk premium. An increase of γ_{π} from 3 (baseline) to 10 leads to a decrease of risk premia of all maturities. For example, the 10-period risk premium decreases from 1.8 to only 0.5 basis points. The results are plotted in figure 5.

Role of habit formation: does habit formation preferences plays a role in increasing the size of risk premia? To answer this question, I change the habit strength parameter from the baseline value of 0.65 to a high level of 0.95 with others parameters set at their baseline values. Habit formation preferences are known to positively magnify the size of risk premium in endowment economy. In this paper, the habit effect on risk premia depends on whether the capital stock is fix or is allowed to vary over time. Figure 6 and 7 plot the risk premium as functions of the habit strength parameter η without capital adjustment cost and with a very high capital adjustment cost respectively. Figure 6 shows that when the agent can freely adjust the capital stock, increasing the habit strengh parameter has limited impact on the size of risk premia and the result is actually a decreasing effect. However, Figure 7 shows that, when the adjustment cost in capital stock is set high enough to fix the capital stock, increasing the habit strength paramater has a large impact on the size of risk premia.

This result is important because habit formation preferences are usually found to magnify risk premia in DSGE models. The increasing effect of the habit strength on the size of risk premia is consistent with other papers in Ravenna and Seppala (2005); Rudebusch *et al* (2006) where the capital stock has been fixed. The reason of this result is as follow. In endowment economies habit preferences significantly magnify the size of risk premia because a risk-averse investor with these preferences, fears more capital losses than an investor with standard preferences. This is because a habit preferences agent cares about not only the level of consumption but the consumption relative to a reference level increasing the agent risk aversion in bad times. As resources can only be intertemporally shifted through financial market, a risk averse investor will claim a larger compensation to hold a long-term bond instead of rolling over short-term bonds. In production economies as this model, there are many alternative channels available for resources transfer. For example, investors can save by accumulating capital through investment or they can even offset bad income shocks by working more. These additional channels for consumption smoothing weaken the habit strength effect on the size of risk premia in production economies. The more channels are available for consumption smoothing, the less important will the habit parameter has on risk premia. Thus, when the capital stock is fixed, the consumer has now less channels for consumption smoothing and then the habit strength will have more impact on risk premia.

The prices of risk associated with the shocks $rp_\ell^a, rp_\ell^{\tilde{z}}, rp_\ell^{mp}$ in (35) are functions of structural parameters. I find that the habit strength parameter η effect on these prices depends also upon the capital adjustment cost parameter. When the adjustment cost parameter is set to $\varphi = 0$, the preferences associated price per unit of risk also decreases as the habit strength increases. When φ is set at very high level, the preferences shock price per unit of risk is increasing with the habit strength parameter.

7 Conclusion

I study in this work the term structure of nominal bonds interest rates in a New-keynesian framework with habit formation preferences, adjustment cost in capital stock. The model features three shocks: preferences shock, technology shock and monetary policy shock. Focus has been on the effect of key structural parameters on risk premia; monetary policy effect on risk premia and shock contribution to the determination of the model implied risk premia.

The model is calibrated to the U.S. economy at a quarterly frequency. Results show that the calibrated second-order approximate solution delivers sizeable and positive risk-premia and an upward sloping average term structure of interest rates. I find that when the productive capital stock is fixed, a higher habit formation parameter significantly increases the risk premium. However when the capital stock is allowed to vary, increases in habit strength decrease risk premium. Moreover, monetary policy has a huge impact on interest rates premia. Especially, an agressive monetary policy leads to decreases in risk premia as it leads to more stable inflation and then to decreases in inflation risk. In terms of contribution of the three shocks, I find that in the benchmark model, preference shock contributes far more to the risk premiums followed by productivity shock and the least important.

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1 Appendix A

Figure 1: average term structure

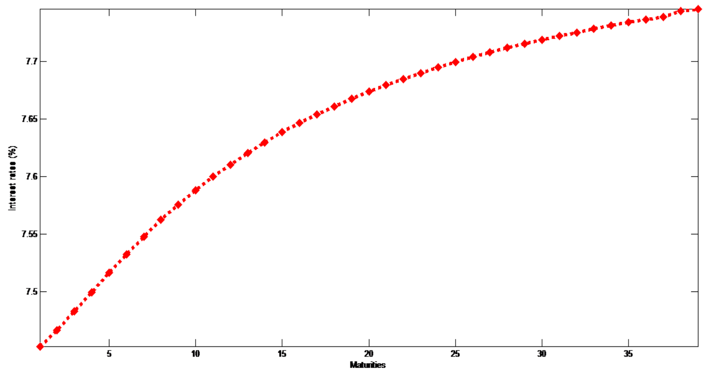


Figure 2: Impulses Responses to Monetary Policy Shock

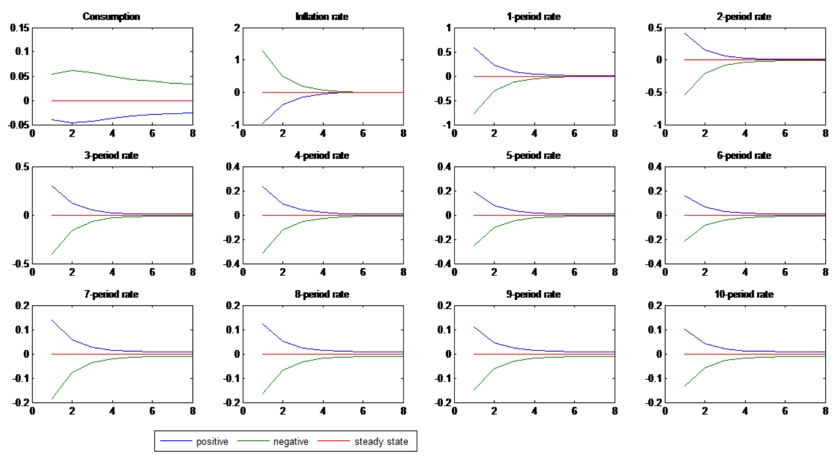


Figure 3: Impulses Responses to Productivity Shock

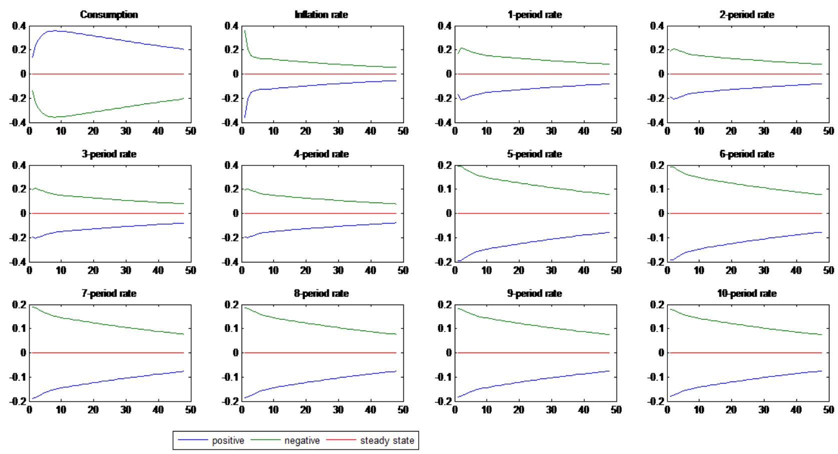


Figure 4: Impulses Responses to Preference Shock

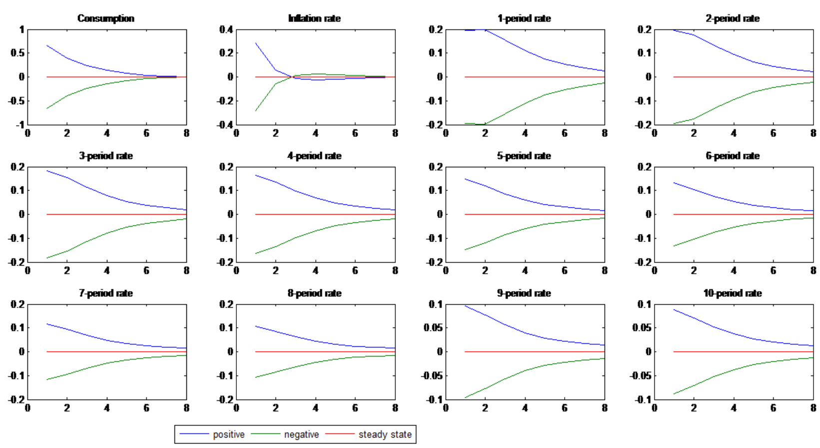


Figure 5: Effect of Monetary Action on Risk Premia

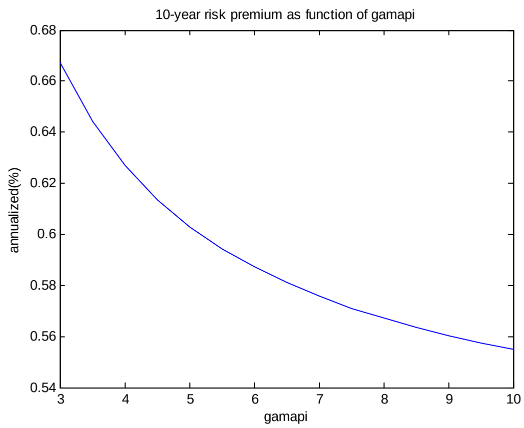
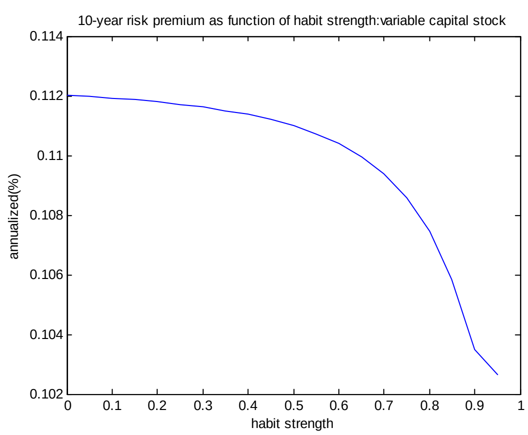
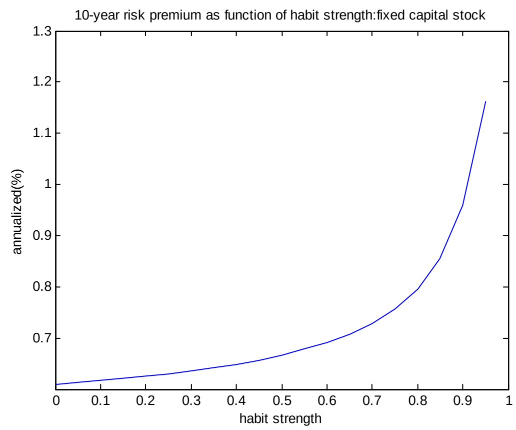


Figure 6: Effect of Habit Strength on Risk Premia: No Capital Adjusment Cost



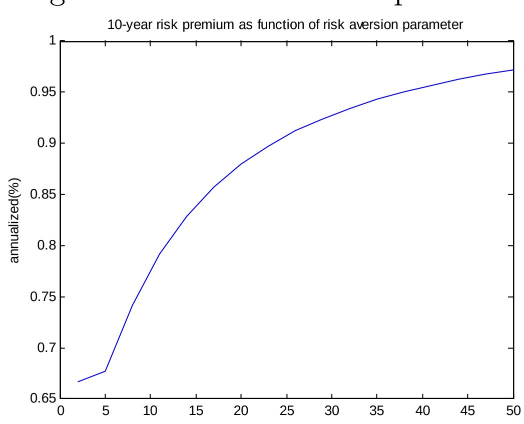
all parameter are baseline and capital adjustment is zero(phi=0)

Figure 7: Effect of Habit Strength on Risk Premia:High Capital Adjusment Costl



All paramaters are baseline except the capital adjustment cost parameter (phi). phi has been set high enough to imply fixed capital stock

Figure 8: Effect of Consumption Curvature parameter on Risk Premia



2 Appendix B: Recursive formulation of auxiliary variables

From the price index equation we get the gross inflation rate dividing each side by P_{t-1}

$$\frac{P_t}{P_{t-1}} = \left[\theta_p + (1-\theta_p)\frac{P_t^{*1-\theta}}{P_{t-1}^{1-\theta}}\right]^{\frac{1}{1-\theta}}$$

where $\frac{P_t^{*1-\theta}}{P_{t-1}^{1-\theta}} = \left[\frac{\theta}{\theta-1}(1+\pi_t)\frac{\widetilde{V_t}}{\widetilde{J_t}}\right]^{1-\theta}$ and $\widetilde{V_t} = \frac{V_t}{P_t^{\theta}}$, $\widetilde{J_t} = \frac{J_t}{P_t^{\theta-1}}$

To get the recursive formulaes of $\widetilde{V_t}$ and $\widetilde{J_t}$ divide equations (**??**) by P_t^{θ} and equation (**??**) by $P_t^{\theta-1}$:

$$\widetilde{V_t} = \theta_p \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\theta}\widetilde{V_{t+1}}\right] + mc_t y_t \tag{1}$$

$$\widetilde{J_t} = \theta_p \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\theta-1}\widetilde{J_{t+1}}\right] + y_t \tag{2}$$