Monetary Policy in an Economy with High Unemployment

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Abstract

This paper uses a New Keynesian DSGE model with unemployment to analyse the behaviour of unemployment and inflation in response to productivity and monetary shocks in an economy with high level of structural unemployment. We conduct a comparative study of four different scenarios in terms of fluidity of the labour market and level of steady state unemployment. The labour market tightness index appears crucial in the design of the four scenarios as it defines how sclerotic the labour market is. The results show that the more fluid is the labour market, the more volatile is unemployment when productivity shock strikes; and the higher the response in unemployment if its steady state value is high when we simulate a monetary shock. Inflation on the other hand is more responsive when the steady state unemployment is low in response to a monetary shock. Also, the more rigid is wage, the larger the effect of productivity on inflation. Finally, the optimised Taylor rule suggests that more attention should be given to unemployment.

1 Introduction

South Africa's official unemployment rate has been around the 25% mark for more than 20 years. This represents the single most important characteristics of the South African economy and the most pressing problem in policy making. Although there is a significant literature looking at the unemployment issue from the point of view of the functionning of the labour market (), there is no attempt in the literature and in the policy making to evaluate what is the effect of high structural unemployment in the trasmission mechanism of macroeconomic policies in general and monetary policy in particular. This paper aims at filling this gap in the literature. We estimate a New Keynesian DSGE model with unemployment of the South African economy and we evaluate how labour market structure affects the transmission of monetary policy.

Over the past few years there has been a growing interest in including the possibility of unemployment in the standard DSGE framework (see for example Gali, 2010, Blanchard & Gali, 2010). However, very few of these studies account for high level of steady state unemployment; which is quite surprising given unemployment is an important indicator of resource utilisation in a macro economy (Gali, Smets & Wouters, 2010).

Blanchard and Gali (2010) built a model with unemployment and labour market frictions that combines standard preferences, real wage rigidities and price staggering. They derive three policies to respond efficiently to productivity shocks. These policies include: constant unemployment, strict inflation targeting and an optimal monetary policy that combines the better of the first two. They compare the quantitative effects of two types of labour market namely the US labour market (which is fluid with low unemployment duration) and the continental Europe labour market (which is sclerotic with high unemployment duration). They find that productivity shocks have no effect on unemployment in the constrained efficient allocation. The same conclusion holds when labour market frictions are introduced to the model. Also, the authors conclude that in fluid labour markets, labour market tightness fluctuates closely with unemployment whereas in sclerotic labour markets, it varies closely with the change in unemployment. This result implies different responses to productivity shocks depending on the type of labour market. For instance, under inflation targeting policy they find that productivity shocks have more persistent effects in a sclerotic than in a fluid labour market. Finally, Blanchard and Gali (2010) show that inflation stabilization is not the best policy to follow in a framework with labour market frictions and real wage rigidities since this policy may lead to inefficient, large, and persistent movements in unemployment in response to productivity shocks. The persistence is even larger in sclerotic labour markets. Therefore, they suggest an optimal monetary policy that accommodates inflation and limits the size of fluctuations in unemployment.

Faia (2009) combines labour market frictions and price stickiness and derives an optimal monetary policy that efficiently deals with the trade-off between inflation and unemployment. She defines three sources of inefficiency in the economy. Firstly, the monopolistic competition only allows for an inefficiently low level of output to be produced and sold at a price that is not at its equilibrium level. Secondly, the cost of adjusting prices causes output to be below its efficient level. Finally, the presence of externalities squeezes the labour market and causes inefficient fluctuations in unemployment. She finds that to counter an increase in productivity, central bankers should reduce inflation in order to increase the demand which causes firms to hire more, therefore reducing the unemployment rate. She concludes that surprisingly, the optimal monetary policy diverges from stable inflation in an attempt to counter productivity and government expenditure shocks. This deviation is increasing in the bargaining power of workers. The author also argues that when workers' bargaining power is above its equilibrium value, firms have little incentive to post vacancies therefore pushing equilibrium unemployment above its social optimum.

Thomas (2008) believes that because of the Walrasian labour market assumption inherited from the real business cycle literature, the New Keynesian model in its simple form does not account for unemployment. Therefore, he introduces imperfections to the labour market in the form of search and matching frictions. Given this framework, he also derives the optimal monetary policy response. The emphasis of his study seems however more on explaining the causes of labour market frictions than actually explaining unemployment and its behaviour. He argues that with the search frictions present in the labour market, unemployed individuals find it hard to get a job and vacancies cannot therefore be easily filled. The author finds that the best policy response when the wages are flexible is achieved by keeping prices constant (zero inflation policy). Doing so will eliminate the discretionary effects of price staggering. However a framework with flexible wages is rather fictitious. Economies nowadays face nominal wage rigidities that create two kinds of distortions. First, real wage rigidity creates inefficient job creation and therefore inefficient unemployment fluctuations; and second, wage dispersion across firms creates inefficient dispersion in hiring rates. He then concludes that given this framework, it is no longer optimal to pursue a zero inflation policy. Instead, the central banker should use price inflation as a tool to bring real wage to its flexible level.

Gali (2010) justifies this absence of unemployment in the previous literature by the fact that it has never been the main focus when central bankers set interest rates; therefore, there was no need to explicitly explain such a phenomenon. The objectives of his study are: to present a clear description of a model combining labour market frictions and nominal rigidities; and to illustrate how such a model can answer questions regarding the interaction between labour market frictions and nominal rigidities. The study also tries to answer the following questions: how can labour market frictions interfere with the design of monetary policy? Should the monetary authority care at all about unemployment when setting interest rates?

Other authors have studied unemployment and its relevance in the design of monetary policy using different approaches. For instance, Shimer (2005) and Hall (2005) combined labour market frictions and real wage rigidity but they both assumed linear preferences. Although Gertler and Trigari (2009) assumed standard preferences (quadratic utility function), they only associate that assumption with labour market frictions and real wage rigidities. Just like Blanchard and Gali (2010), some authors (Christoffel & Linzert, 2005; Krause & Lubik, 2007) combined standard preferences, labour market frictions, real wage rigidity and price staggering. However, they used more complex models.

We organise the rest of the paper as follows:.In the next section we sets up the model focusing in particular of the role of labour market frictions in the model in the form of time varying cost per hire and nominal wage rigidities. Section 3 solves the problem and defines three equilibriums. We determine the first one by assuming a benevolent social planner who internalises the effects of the changes in the labour market tightness on hiring costs and on the resource constraint. The second one is consistent with Nash-bargained wages whereas the third equilibrium is consistent with real wage rigidities. All three cases come down to the same conclusion: the level of employment is invariant to productivity shocks. Section 4 determines the Philips curve relation between inflation and unemployment implied by the proposed study and then sets up the equilibrium dynamics. We also calibrate the model in this section. Section 5 analyses quantitatively the responses of inflation and unemployment to productivity and monetary shocks, and we conduct the welfare analysis in section 6. Section 7 concludes.

2 The Model

The model of Blanchard and Gali (2010) follows the typical structure of a standard New Keynesian model. Thus, the economy is composed by a large number of identical household, monopolistically competitive firms in the final good sector, subjected to price rigidities a la Calvo, and intermediate good firms operating in a perfectly competitive market which demand labour in a

2.1 Household

We assume standard preferences and a large number of identical households. Each one is composed of a continuum of members represented by the unit interval. The household maximises the objective function given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t,\ L_t) \tag{1}$$

where $\beta \in [0,1]$ is the discount factor, $C_t \equiv \left(\int_0^1 C_t\left(i\right)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$ is the quantity consumed of final goods. ? denotes price elasticity. Let L_t be the index of the total time that household members allocate to labour and market activities:

$$L_t = N_t + \psi U_t \tag{2}$$

where N_t and U_t respectively denote the fraction of household members who are employed and unemployed (but looking for a job). ψ represents the marginal disutility generated by an unemployed member relative to an employed one. Since household members supply labour to intermediate goods firms participating in perfect competition, there is no involuntary unemployment. Thus, $L_t = N_t$ for all t.

Also,

$$0 \leq N_t \leq 1 \tag{3}$$

The household's utility function is of the following form:

$$U\left(C_t,\ N_t\right) \equiv logC_t - \frac{\chi}{1+\phi} {N_t}^{1+\phi} \tag{4}$$

The household faces the following budget constraint:

$$\int_0^1 P_t\left(i\right) C_t\left(i\right) di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t\left(j\right) N_t\left(j\right) dj + \Pi_t \tag{5}$$

Where $P_t(i)$ is the price of good i, $W_t(j)$ is the nominal wage paid by firm j, B_t denotes purchases of one-period bonds at a price Q_t, and Π_t represents a lump-sum component of income which may include dividends from ownership of firms or lump-sum taxes. Note consumption expenditures can be rewritten as $\int_0^1 P_t(i) C_t(i) di = P_t C_t$ where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$ is the price of final goods.

Optimal demand for each good is given by:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{6}$$

and the inter-temporal optimal condition takes the form:

$$Q_t = \beta E_t \left\{ \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\} \tag{7}$$

which we also denote as the relevant stochastic discount factor.

2.2 The firms

There are two types of firms in the economy. Firms producing final goods face a monopolistic competition. They do not use labour as input and are subject to nominal rigidities. Intermediate goods firms on the other hand operate in a perfectly competitive environment and use labour as input.

Final goods firms

There is a continuum of final goods firms indexed by $i \in [0,1]$, each producing a differentiated final good. They all have access to the same technology:

$$Y_t(i) = X_t(i) \tag{8}$$

where $X_t(i)$ denotes the single intermediate good used by firm i as an input.

We set prices following Calvo (1983). Each period, only a randomly selected fraction $1-\theta$ of final goods firms gets to change their prices. For the rest of the final goods producers measured by θ their price remains at the same level. Parameter $\theta \in [0,1]$ can be interpreted as an index of price rigidities. Aggregate price level satisfies the following:

$$P_t = \left((1-\theta)\left(P_t^*\right)^{1-\epsilon} + \theta (P_{t-1})^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{9}$$

where P_t^* is the price newly set by a final goods firm at time t.

The optimal price setting rule for a firm resetting prices in period t is given by:

$$E_t \left\{ \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t} (P_t^* - \mathcal{M} P_{t+k} MC_{t+k} \right\} = 0 \tag{10}$$

in which $Y_{t+k|t}$ denotes the level of output in period $t+k$ for a firm resetting price in period t, $\mathcal{M} \equiv \epsilon/(\epsilon-1)$ represents the gross mark up and MC_t is the real marginal cost for final goods firms.

Intermediate good firms and labour market frictions

We assume a continuum of identical, perfectly competitive firms, represented by the unit interval and indexed by j that produces intermediate goods. All firms have access to the same production function of the form:

$$X_t\,(i) = A_t N_t(j) \tag{11}$$

Variable A_t represents the state of technology, which is assumed to be common across firms and varies exogenously over time. More precisely, $a_t \equiv \log A_t$ follows an AR process with autoregressive coefficient ρ_a and variance σ_a^2.

Employment in firm j evolves according to:

$$N_t\,(j) = (1-\delta)\,N_{t-1}\,(j) + H_t(j) \tag{12}$$

where $\delta \in (0,1)$ is an exogenous separation rate, and $H_t(j)$ represents the measure of workers hired by firm j in period t. Note that new hires start working in the same period they are hired. This assumption deviates from the standard one in the search and matching model in which a one period lag before a hired worker becomes productive is required. However, it is consistent with conventional business cycle models in which employment is not a predetermined variable.

There is a pool of jobless individuals (available for hire) in the beginning of period t given by U_t (Blanchard & Gali, 2010). At all time, individuals are either employed or willing to work, depending on the conditions prevailing in the labour market. The assumption of full participation therefore holds. Thus,

$$U_t = 1 - N_{t-1} + \delta N_{t-1} = 1 - (1-\delta)N_{t-1} \tag{13}$$

Among those unemployed at the beginning of period t, a measure $H_t \equiv \int_0^1 H_t\,(j)\,dj$ are hired.

Aggregate hiring evolves according to:

$$H_t = N_t - (1-\delta)N_{t-1} \tag{14}$$

where $N_t \equiv \int_0^1 N_t\,(j)\,dj$ represents aggregate employment.

We now introduce labour market frictions to the model in the form of cost per hire represented by G_t which we assume to be exogenous to individual firms. However it depends on aggregate factors, including the labour market tightness index represented by $x_t \in [0,1]$ and given by:

$$x_t \equiv \frac{H_t}{U_t} \tag{15}$$

The aforementioned simply means that only workers in the unemployment pool at the beginning of the period can be hired ($H_t \leq U_t$). Also known as job finding rate, x_t captures the probability of getting hired in period t.

It follows that $G_t = G(x_t)$. Note that hiring costs for an individual firm are given by $G_t H_t(j)$, expressed in terms of the CES bundle of goods. G_t is increasing in labour market tightness and more formally:

$$G_t = A_t B x_t^{\alpha} \tag{16}$$

where $\alpha \geq 0$ and B is a positive constant. For convenience, let $g_t \equiv B x_t^{\alpha}$. It follows that:

$$G_t = A_t g_t.$$

This formulation means that vacancies are immediately filled by paying the hiring cost; which diverges from the Diamond-Mortensen-Pissarides search and matching model of unemployment in which the hiring cost is uncertain. Since the aim of the proposed study is not explaining vacancies, the approach we choose will therefore be the one by Blanchard and Gali (2010) which is, as they pointed it out, very simple.

To close up with this section, it is important to define an alternative measure of unemployment, given by u_t, which is the fraction of the population who are left without a job after hiring has taken place in period t. It is written as follows

$$u_t = U_t - H_t = 1 - N_t \tag{17}$$

2.3 The equilibrium

The social planner equilibrium

We assume a benevolent social planner who solves the problem facing technological constraints and labour market frictions present in the decentralized economy. He internalises the effects of changes in the labour market tightness on hiring costs and the resource constraint.

Since there is symmetry in preferences and technology, efficiency requires that identical quantities of goods be consumed and produced, meaning $C_t(i) = C_t$ for all $i \in [0, 1]$. Also, labour market participation has no cost, but instead it has a social benefit since it decreases hiring costs. The social planner always chooses an allocation with full participation. This necessarily does not imply full employment since both a disutility and increases in hiring costs come as a result of higher employment.

The social planner therefore maximises 1 subject to 3 and the aggregate resource constraint given by:

$$C_t = A_t(N_t - B x_t^{\alpha} H_t) \tag{18}$$

As shown in the appendix, the optimality condition for the social planner's problem is given by:

$$\chi C_t N_t^{?} \leq A_t - (1+\alpha) A_t B x_t^{\alpha} + \beta(1-\delta) E_t \left\{ \frac{C_t}{C_{t+1}} A_{t+1} B x_{t+1}^{\alpha} (1 + \alpha(1 - x_{t+1})) \right\} \tag{19}$$

Thus, the marginal rate of substitution between labour and consumption (on the left hand side) is equal to or less than the marginal rate of transformation between the same labour and consumption (on the right hand side). The marginal rate of transformation has two distinct terms. The first one represents the additional output generated by a marginal employed worker whereas the second term captures the savings in hiring costs resulting from the reduced hiring needs in period $t+1$.

First off, we consider a solution in which labour market frictions are absent ($B = 0$). It follows that $C_t = A_t N_t$ and the equilibrium condition becomes:

$$\chi N^{1+\phi} = 1 \tag{20}$$

if $\chi \geq 1$, or $N_t = 1$ if $\chi < 1$. Either way, the level of employment is invariant to productivity shocks. Since there is no capital accumulation, consumption increases in proportion to productivity. This increase in consumption leads to an income effect that offsets exactly the substitution effect, hence the invariance of the level of employment to productivity shocks.

The solution when labour market frictions are present ($B > 0$) involves a constant job finding rate denoted by x^*. Assuming positive unemployment in equilibrium, this solution is implicitly given by:

$$\left(1 - \delta B x^{\alpha}\right) \chi N\left(x\right)^{1+\phi} = 1 - \left(1 - \beta\left(1 - \delta\right)\right)\left(1 + \alpha\right) B x^{\alpha} - \beta(1-\delta)\alpha B x^{1+\alpha} \tag{21}$$

where $N\left(x\right) \equiv x/(\delta + \left(1 - \delta\right) x)$ is the level of employment given x. The constant unemployment rate implied by the constrained-efficient allocation is thus defined as follows:

$$u = \frac{\delta(1 - x^*)}{\delta\left(1 - x^*\right) + x^*} \tag{22}$$

The new levels of consumption and output are then given by $C_t^* = A_t N^*(1 - \delta B x^{*\alpha})$ and $Y_t^* = A_t N^*$.

Again, employment is invariant to productivity shocks. This time, the reason behind this result is that at the same level of unemployment, both the marginal rate of substitution and the social marginal rate of transformation increase in the same proportion as productivity. Note that in the presence of capital accumulation, this invariance of employment to productivity shocks would no longer hold.

Equilibrium under flexible prices and wage determination

Price setting

P_t denotes the price index associated with C_t, P_t^I is the price of the intermediate good and W_t represents the real wage in terms of the bundle of final goods.

Intermediate goods producers are price takers and their profit maximisation suggest that for all t, the real marginal revenue product of labour equals the real marginal cost. Thus,

$$\left(\frac{P_t^I}{P_t}\right) A_t = W_t + G_t - \beta(1-\delta)E_t\left(\frac{C_t}{C_{t+1}}G_{t+1}\right) \tag{23}$$

On the other hand, profit maximisation by final goods producers requires $P_t = \mathcal{M}P_t^I$ for all t.

Using 23 and reorganising gives:

$$Bx_t^\alpha = \left(\frac{1}{\mathcal{M}} - \frac{W_t}{A_t}\right) + \beta(1-\delta)E_t\left(\frac{C_t}{C_{t+1}}\frac{A_{t+1}}{A_t}Bx_{t+1}^\alpha\right) \tag{24}$$

Solving forward and the result shows that labour market tightness depends on the expected discounted stream of marginal profits generated by an additional hire. Marginal profit depends, in turn, on the ratio of the wage to productivity.

Determination of Wage

The presence of labour market frictions generates a surplus associated with established employment relationships. The wage determines how that surplus is divided between workers and firms. In this section we present two ways of determining wage namely flexible and sticky wages. Under flexible wages, all wages are renegotiated and adjusted every period. On the other hand under sticky wages, only a fraction of firms can adjust their nominal wages in any given period.

Flexible wages

We determine flexible wages using Nash bargaining techniques. Each firm negotiates with its workers over their individual compensation. The value of an employed member to a household is given by:

$$\mathcal{V}_t^N = W_t - \chi C_t N_t^\phi + \beta E_t\left\{\frac{C_t}{C_{t+1}}\left[\left(1-\delta\left(1-x_{t+1}\right)\right)\mathcal{V}_{t+1}^N + \delta(1-x_{t+1})\ \mathcal{V}_{t+1}^U\right]\right\} \tag{25}$$

$\mathcal{V}_t^U$ is the value of an unemployed member to a household and is given by:

$$\mathcal{V}_t^U = \beta E_t\left\{\frac{C_t}{C_{t+1}}\left[x_{t+1}\ \mathcal{V}_{t+1}^N + (1-x_{t+1})\ \mathcal{V}_{t+1}^U\right]\right\} \tag{26}$$

From an established employment relationship, the household's surplus is given by $\mathcal{S}_t^H \equiv \mathcal{V}_t^N - \mathcal{V}_t^U$ and can be written as:

$$\mathcal{S}_t^H = W_t - \chi C_t N_t^\phi + \beta(1-\delta)E_t\left\{\frac{C_t}{C_{t+1}}(1-x_{t+1})\mathcal{S}_{t+1}^H\right\} \tag{27}$$

On the other hand and again from an established employment relationship, the firm's surplus, represented by $\mathcal{S}_t^F$, is given by:

$$\mathcal{S}_t^F = A_t B x_t^\alpha = G_t \tag{28}$$

meaning any currently employed individual can be immediately substituted with an unemployed one just by paying the hiring cost.

The Nash bargain must satisfy:

$$\mathcal{S}_t^H = \vartheta \mathcal{S}_t^F \tag{29}$$

where ϑ is the relative bargaining power of workers. By combining this condition with 27 and 28, we obtain the following wage schedule:

$$W_t = \chi C_t N_t^{\phi} + \vartheta \left(A_t B x_t^{\alpha} - \beta(1-\delta) E_t \left\{ \frac{C_t}{C_{t+1}} (1 - x_{t+1}) A_t B x_{t+1}^{\alpha} \right\} \right) \tag{30}$$

Therefore, given that workers have some bargaining power ($\vartheta > 0$) and that labour market frictions are present ($B > 0$), the bargained wage equals to the marginal rate of substitution plus an additional term reflecting labour market conditions. This additional term is an increasing function of current labour market tightness (given that when associated with an existing relationship, it raises the firm's surplus) but is a decreasing function of expected future hiring costs ($G_{t+1} = A_t B x_{t+1}^{\alpha}$) and the probability of not finding a job if unemployed next period given by $1 - x_{t+1}$ (since these two terms lower wage today by increasing the continuation value to an employed worker).

By combining 24 (which gives the wage consistent with the price setting) and 30 (giving the wage consistent with Nash bargaining), we obtain the following new equilibrium:

$$\chi C_t N_t^{?} = \frac{A_t}{\mathcal{M}} - (1+\vartheta) A_t B x_t^{\alpha} + \beta(1-\delta) E_t \left\{ \frac{C_t}{C_{t+1}} A_t B x_{t+1}^{\alpha} (1 + \vartheta (1 - x_{t+1})) \right\} \tag{31}$$

Once again the equilibrium implies a constant job finding rate x, implicitly given by:

$$(1 - \delta B x^{\alpha}) \chi N(x)^{1+\phi} = \frac{1}{\mathcal{M}} - (1 - \beta(1-\delta))(1+\vartheta) B x^{\alpha} - \beta(1-\delta) ? B x^{1+\alpha}$$

This constant job finding rate, in turn, implies a constant unemployment rate:

$$u = \frac{\delta(1-x)}{\delta(1-x) + x} \tag{32}$$

This time however, consumption, output and real wage fluctuate in proportion to productivity.

The real wage is given by:

$$W_t = \left(\frac{1}{\mathcal{M}} - (1 - \beta(1-\delta)) B x^{\alpha} \right) A_t \tag{33}$$

Note that this unemployment rate differs from the social planner's rate under the equilibrium with Nash-bargained wages. The two unemployment rates coincide when the relative bargaining power of workers matches the elasticity of hiring costs relative to the labour market tightness index, and the effective market power by final goods firms is absent. In other words, $\vartheta = \alpha$ and $\mathcal{M} = 1$. No matter the difference between these two unemployment rates, they both share one property namely, their invariance to productivity shocks.

According to this flexible design, wages are highly responsive to productivity movements. However empirical studies show a different outcome. This result has led authors (Shimer, 2005, Hall, 2005) to account for real wage rigidities to explain small movements in the wage that match large movements in unemployment.

Real wage rigidities

Formalising real wage rigidities remains a question open to research. Thus, for simplicity, let's assume like Blanchard and Gali (2010) a wage schedule of the form:

$$W_t = \Theta A_t^{1-\gamma} \tag{34}$$

in which $\gamma \in [0,1]$ is an index of real wage rigidities, and Θ is a positive constant which we assume to take the value $\Theta \equiv \left(\left(\frac{1}{\mathcal{M}}\right) - (1-\beta(1-\delta))\, Bx^{\alpha}\right) A^{\gamma}$, with A representing the unconditional mean of A_t. Note that for $\gamma = 0$ (flexible wages) this wage schedule equals to the Nash-bargained wage.

By combining 34 with 24, we derive the last equilibrium condition, namely the equilibrium consistent with real wage rigidities, which is given by:

$$\Theta A_t^{-\gamma} = \frac{1}{\mathcal{M}} - Bx_t^{\alpha} + \beta(1-\delta)E_t\left\{\frac{C_t}{C_{t+1}}\frac{A_{t+1}}{A_t}Bx_{t+1}^{\alpha}\right\} \tag{35}$$

Rearranging and solving forward:

$$Bx_t^{\alpha} = \sum_{k=0}^{\infty}(\beta(1-\delta))^k E_t\left\{\frac{C_t}{C_{t+1}}\frac{A_{t+1}}{A_t}\left(\frac{1}{\mathcal{M}} - \Theta A_{t+k}^{-\gamma}\right)\right\} \tag{36}$$

This equation highlights the importance of the role played by labour market tightness in an economy with labour market frictions and real wage rigidities. Given that wages are not fully flexible, labour market tightness and, by implication, movements in employment and unemployment, depend on current and anticipated productivity.

2.4 Log Linearization

In order to illustrate the equilibrium dynamics we first define the real marginal cost which we assume to evolve according to P_t^I/P_t. Combining the profit maximisation condition of intermediate goods producers given by 23 with the wage schedule in equation 34 gives the following setting for real marginal cost:

$$MC_t = \Theta A_t^{-\gamma} + Bx_t^{\alpha} - \beta(1-\delta)E_t\left\{\frac{C_t}{C_{t+1}}\frac{A_{t+1}}{A_t}Bx_{t+1}^{\alpha}\right\} \tag{37}$$

One can easily detect that real marginal cost depends on labour market frictions (captured by hiring cost parameters B and α) and on real wage rigidities (measured by the rigidity index γ).

Lower case variables with hats represent log deviations of the corresponding upper case variables from their steady state values.

After log-linearizing equations 9 and 10 around a zero inflation steady state (Gali & Gertler, 1999), the following expression for inflation is given:

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda \widehat{mc}_t \tag{38}$$

in which $\lambda \equiv (1 - \beta\theta)(1 - \theta)/\theta$

Log linearizing and rearranging equation 37, real marginal cost takes the following form:

$$\widehat{mc}_t = \alpha g \mathcal{M} \hat{x}_t - \beta (1 - \delta) g \mathcal{M} E_t \{(\hat{c}_t - \hat{a}_t) - (\hat{c}_{t+1} - \hat{a}_{t+1}) + \alpha \hat{x}_{t+1}\} - \Phi \gamma \hat{a}_t \tag{39}$$

in which $\Phi \equiv \frac{\mathcal{M}W}{A} = 1 - (1 - \beta(1 - \delta)) g \mathcal{M} < 1$.

It then follows that real marginal cost is positively related to labour market tightness and negatively (given $\gamma > 0$) to productivity. The more rigid is real wage, or the more persistent the productivity process, the larger the effect of productivity on real marginal cost (and in turn, on inflation).

We derive an expression for labour market tightness as a function of current and lagged employment from equation 15 which takes the following form:

$$\delta \hat{x}_t = \hat{n}_t - (1 - \delta)(1 - x)\hat{n}_{t-1} \tag{40}$$

From equation 18, an expression for consumption is obtained:

$$\hat{c}_t = \hat{a}_t + \frac{1 - g}{1 - \delta g}\hat{n}_t + \frac{g(1 - \delta)}{1 - \delta g}\hat{n}_{t-1} - \frac{\alpha g}{1 - \delta g}\delta \hat{x}_t \tag{41}$$

Finally, from the consumer's first order conditions, we obtain the following:

$$\hat{c}_t = E_t \{\hat{c}_{t+1}\} - (i_t - E_t \{\pi_{t+1}\} - \rho) \tag{42}$$

in which $\rho \equiv -log\beta$.

We now move on to derive the Philips curve relation between inflation and unemployment implied by the framework of the proposed study. Substituting 40 in 41gives:

$$\hat{c}_t = \hat{a}_t + \xi_0 \hat{n}_t + \xi_1 \hat{n}_{t-1} \tag{43}$$

in which $\xi_0 \equiv (1 - g(1 + \alpha))/(1 - \delta g)$ and $\xi_1 \equiv (g(1 - \delta))(1 + \alpha(1 - x))/(1 - \delta g)$.

Substituting this expression, along with 40, in 39 gives this new expression for real marginal cost:

$$\widehat{mc}_t = h_0 \hat{n}_t + h_L \hat{n}_{t-1} + h_F E_t \{\hat{n}_{t+1}\} - \Phi \gamma \hat{a}_t \tag{44}$$

in which

$$h_0 \equiv \left(\frac{\alpha g \mathcal{M}}{\delta}\right)\left(1 + \beta(1 - \delta)^2 (1 - x)\right) + \beta (1 - \delta) g \mathcal{M}(\xi_1 - \xi_0)$$

$$h_L \equiv -(\alpha g\mathcal{M}/\delta)(1-\delta)(1-x) - \beta(1-\delta)g\mathcal{M}\xi_1$$
$$h_F \equiv -\beta(1-\delta)g\mathcal{M}((\alpha/\delta)-\xi_0)$$

By replacing this expression in equation 38, and given $\hat{u}_t = -(1-u)\hat{n}_t$, the following Philips curve giving the relation between inflation and unemployment is obtained:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \kappa_0\hat{u}_t + \kappa_L\hat{u}_{t-1} + \kappa_F E_t\{\hat{u}_{t+1}\} - \lambda\Phi\gamma\hat{a}_t \tag{45}$$

in which $\kappa_0 \equiv \lambda h_0/(1-u)$, $\kappa_L \equiv -\lambda h_L/(1-u)$ and $\kappa_F \equiv -\lambda h_F/(1-u)$. This Philips curve highlights the negative relationship between inflation and both the level and the change in the unemployment rate.

Again, using the relation between employment and unemployment given by $\hat{u}_t = -(1-u)\hat{n}_t$, equation (40) becomes:

$$(1-u)\,\delta\hat{x}_t = -\hat{u}_t + (1-x)(1-\delta)\hat{u}_{t-1} \tag{46}$$

which gives the relation between labour market tightness and both current and lagged unemployment rates. It plays an important role in the design of the four scenarios implied by the proposed study. To start up, we consider two labour markets. The first one is characterised by labour market high flows (implying high values of δ and x) and low unemployment duration. The other is considered to have low flows (low level of δ and x) and relatively high steady state unemployment. We define the first labour market as fluid whereas the second one is more rigid. The fluid labour market has a small $(1-x)(1-\delta)$ given in 46 while this value is larger for the rigid labour market. Thus, relative labour market tightness moves more with the negative of the change in the unemployment rate; consequently, changes in unemployment lead to large relative changes in the flows. On the other hand, in the fluid labour market with low steady state unemployment, changes in unemployment lead to small relative changes in the flows, thus to small relative changes in labour market tightness. The other two scenarios are a fluid labour market with high unemployment duration and a sclerotic labour market with low unemployment duration.

The equilibrium is therefore characterised by the following set of equations

1. The Philips curve relation between inflation and unemployment:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \kappa_0\hat{u}_t + \kappa_L\hat{u}_{t-1} + \kappa_F E_t\{\hat{u}_{t+1}\} - \lambda\Phi\gamma\hat{a}_t$$

2. The relation between unemployment and employment:

$$\hat{u}_t = -(1-u)\hat{n}_t$$

3. The expression of labour market tightness as a function of current and lagged employment:

$$\delta\hat{x}_t = \hat{n}_t - (1-\delta)(1-x)\hat{n}_{t-1}$$

4. The expression for consumption:

$$\hat{c}_t = \hat{a}_t + \frac{1-g}{1-\delta g}\hat{n}_t + \frac{g(1-\delta)}{1-\delta g}\hat{n}_{t-1} - \frac{\alpha g}{1-\delta g}\delta\hat{x}_t$$

5. The first order condition for the consumer:

$$\hat{c}_t = E_t\{\hat{c}_{t+1}\} - (i_t - E_t\{\pi_{t+1}\} - \rho$$

6. The central banker's instrument:

$$i = \rho + \varphi_\pi \pi_t + \varphi_c c_t + \varphi_u u_t$$

3 Calibration and Simulation

Each period corresponds to a quarter. Parameters describing preferences take common values. Thus, $\beta = 0.99$, $\phi = 1$ and $\epsilon = 6$. This implies a value of 1.2 for the mark up. Nakamura and Steinsson (2008) estimate a median price duration between 8 and 12 months. Therefore, $\lambda = 1/12$. Since no hard evidence on the degree of real wage rigidities is existent, we assumed that $\gamma = 0.5$. to α we assign the value of 1. The level of hiring cost takes the following value $B = 0.12$.

Given the relationship between unemployment, labour market tightness and the separation rate $u = \frac{\delta(1-x)}{\delta(1-x)+x}$, we assume four scenarios as explained in section 4.1:

1. Scenario 1 (Rigid-Low): a rigid labour market with low steady state unemployment rate with $u = 0.05$, $x = 0.15$ and $\delta = 0.01$. Here we assume an economy with low steady state unemployment. We define a labour market where it is hard for unemployed individuals to find a job and once they have got it, they hold on to it for a while and do not lose it easily. The flows in the labour market are therefore low.

2. Scenario 2 (Rigid-High): a rigid labour market with high steady state unemployment rate with $u = 0.30$, $x = 0.15$ and $\delta = 0.075$. This scenario differs from the previous one on the level of steady state unemployment which we assume in this scenario to be quite high. Also, the separation rate is only slightly bigger.

3. Scenario 3 (Fluid-Low): a fluid labour market with low steady state unemployment rate with $u = 0.05$, $x = 0.8$ and $\delta = 0.21$. In this scenario, an unemployed individual has a high chance of finding a job but also letting it go is quite easy (compared to scenario 1).

4. Scenario 4 (Fluid-High): a fluid labour market with high steady state unemployment with $u = 0.30$, $x = 0.67$ and $\delta = 0.87$. An unemployed individual in this economy has a relatively high probability of getting hired. However, he would lose the job fairly easy. Although the flows are high in this labour market, a huge amount of individuals in this scenario is left without jobs.

Figure 1: Dynamic Effects of a Productivity Shock on Inflation

In each scenario, we simulate productivity and monetary shocks on the economy. The two shocks are AR processes with autoregressive coefficients of 0.9. The general effects of these shocks are in line with the standard New Keynesian DSGE model (see for instance Gali *et al*, 2010). What is different about this model is the effects on inflation and unemployment at different level of labour market rigidity. Therefore, we are only reporting the quantitative effects of productivity and monetary shocks on these two variables. We also assume that the central banker uses a simple Taylor rule with elasticity parameters taking the following standard values $\varphi_\pi = 1.5$, $\varphi_c = 0.5$ and $\varphi_u = 0$. Finally we report the responses of inflation and unemployment due to a one per cent change in each shock.

3.1 Productivity shock

Figure 1 shows the reaction of inflation to a productivity shock. We find that the steady state level of unemployment along with the type of labour market, play an important role in how inflation responds. In scenario 2 and 3 the responses are practically the same – inflation decreases by about 0.6 per cent on the impact. The most significant drop on the impact (close to 0.9 per cent) occurs in scenario 1 in which inflation displays high response. On the other hand in scenario 4 where unemployment is high, inflation is the least responsive with a decrease of about 0.4 per cent on impact.

Faia (2009) suggests that central bankers should decrease inflation in response to productivity shocks, therefore boosting the demand and resulting into firms hiring more. This result can therefore mean an increase in employment for scenario 2 and 4 characterized by high steady state unemployment rates.

We report the response of unemployment to a productivity shock in figure 2. The result in scenario 2 is consistent with the findings of Blanchard and Gali (2010). They conclude in their study that sclerotic labour markets are

Figure 2: Dynamic Effects of a Productivity Shock on Unemployment

associated with high response in unemployment. Interestingly, we find that the result in scenario 4 diverges from that conclusion. Indeed, scenario 4 which is associated with a fluid labour market instead shows the highest response in unemployment of all four scenarios. Another striking feature in figure 2 is how almost non-volatile unemployment is in scenario 1. It increases by about 0.12 per cent on the impact and slowly converges to the initial level once the shock dies out.

It therefore appears as if unemployment is more responsive the more fluid labour market is. This contradictory result from the conclusion of Blanchard and Gali (2010) is perhaps due to the fact the authors follow an inflation targeting policy. In the proposed study on the other hand, central bankers use a simple Taylor rule.

3.2 Monetary shock

In figure 3, we show the response of inflation to a monetary shock. Inflation is highly responsive in scenario 1 with a drop of about 1.4 per cent on impact. It then converges to the initial level as the shock dies out. The lowest drop (0.2 per cent) is recorded for the calibration of scenario 4. Inflation is barely volatile but takes a very long time to go back to the actual initial value. The degree of response is remarkably larger under scenarios with low level of steady state unemployment. It is therefore appropriate to conclude that according to the calibrations of the model implied by the proposed study, inflation is more volatile when the level of unemployment prevailing in the economy is low.

For variable unemployment on the other hand, the response is higher when the labour market is more fluid as shown in figure 4. On impact, the highest increase is recorded in scenario 4 (1.2 per cent) whereas the lowest increase occurs in scenario 1 (about 0.2 per cent). The discrepancy between scenario 1

Figure 3: Dynamic effects of a monetary shock on inflation

and 2 is probably the most striking feature of figure 4. The increase in scenario 2 (about 0.8 per cent) is about 4 times that in scenario 1. This result implies that when the labour market is rigid, the degree of response of unemployment is positively related to its steady state level. Finally, even though the increase on impact is about the same for scenarios 3 and 4, unemployment converges fairly quicker in the first quarter for scenario 3.

3.3 Welfare Analysis

We now turn to analyse how monetary policy should be conducted efficiently. In other words, how should central bankers react optimally to productivity and monetary shocks in a way that will maximise social welfare? Thus far, we have assumed a standard simple Taylor rule as suggested by the standard DSGE New Keynesian model. The question we are facing now is what are the values of φ_π, φ_c and φ_u corresponding to the optimised Taylor rule that minimise the loss function? The loss function we are referring to is given by:

$$L = \sum_{0}^{\infty} \beta^t \left(\pi_t^2 + u_t^2 \right) \tag{47}$$

We show the result in the table below

Scenario	Rigid-Low	Rigid-High	Fluid-Low	Fluid-High
φ_π	1.66	1.56	1.60	1.56
φ_c	0.33	0.43	0.39	0.43
φ_u	- 0.92	- 0.84	- 1.48	- 0.97

The general picture shows a trade-off between unemployment and consumption with amplitude depending on the scenario. In all four cases, the optimised Taylor rule puts more weight on unemployment and inflation whereas it decreases the weight on consumption. However, scenario 3 is the one that requires

Figure 4: Dynamic effects of a monetary shock on unemployment

the strongest measure on unemployment. This is perhaps due to the fact that because the labour market is fluid in that scenario, unemployment is more responsive to shocks. In scenarios with high steady state unemployment, we find that central bankers should care the same about the way they react on inflation and consumption in both cases. However, they should care a little bit more about unemployment in the scenario with high steady state unemployment and fluid labour market (scenario 4). Therefore, we conclude that the more fluid is labour market, the more central banker should care about unemployment and *vice versa*.

4 Estimation

In this section we present an estimation of the model using South African data to give some idea of the level and the influence of labour market rigidities on the macroeconomic dynamic and on monetary policy effectiveness in controlling inflation.

4.1 Data

We estimate the model on South African data for the sample period 1970Q1-2011Q3 using Bayesian methods (An and Schorfheide, 2006). The four observable variables are GDP, CPI Inflation, Money Market Interest Rate and Total Manufacturing Employment. All the data are sources from the SARB Quaterly Bulleting Database. GDP and Employment are defined in percentage deviation from long term trend, calculated by usual filtering procedures (both HP Filter and Band-Pass filters). The variables used are presented in figure 1. We do not use wage series or explicitly unemployment series (like in Gali, Smets and

Wouters, 2011) to minimize the influence of errors in variable measurement to mini

4.2 Estimation Results

In table 1

5 Conclusions

This paper analysed the influence of labour market institutions on the monetary transmission mechanism in South Africa using a New-Keynesian modelling framework developed by Blanchard and Gali (2010) The analysis suggest that South Africa.owns its high long term unemployment rate to a combination of wage rigidity togheter with an high degree of entry and exit in the labour market, making this the most challanging enviroment to operate monetary policy. In fact, in this enviroment any attempt to control inflation has very significant real impact on the employment conditions, with a veruy significant sacrifice ratio.

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