It is the weight that counts, silly!

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Abstract

Background

The Great Financial Crisis of 2008 changed the way in which investment management institutions viewed investments. Recent trends suggest that investors are no longer prepared to pay high active management fees to only receive beta type performance. Therefore this article focuses on addressing how different weighting schemes to the traditional capital-weighted index perform in terms of risk and return within the South African investment context. We argue that these newly constructed indices could be used as an alternative, superior, benchmark for active managers which would add alpha relative to the traditional capital-weighted index. Investors should, however, be cognisant of the inherent risk the new weighting scheme introduces, which we explain in the report.

Methodology

The study incorporates JSE constituent data for the All Share Index for the period 2008 – 2012. Optimization techniques are used to construct indices based on various criteria such as Minimum Variance, Mean Variance, Maximum Diversity as well as Risk Parity approaches. These newly constructed indices are compared to the traditional capital-weighted FTSE/JSE All Share (ALSI) index, the FTSE/JSE Share Holder Weighted All Share (SWIX) index, capped versions of SWIX as well as a naïve diversification or equally weighted index.

Conclusion

Our analysis indicates that weighting schemes that differ from the traditional capital-weighted index have beneficial characteristics, both in terms of risk and return within the South African investment context. In our opinion, these preferred attributes could be employed to build better building blocks for active asset managers to which alpha could be added.

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1.1 Introduction:

The financial markets started to show some weakness after the deterioration of house prices in late 2007. Poor lending practices led to homeowners owning more than their properties were worth and foreclosing on these properties meant that financial institutions bore the brunt of the defaults. Investors fearing that financial institutions would be unable to repay their loans started to withdraw money from these organisations, which eventually culminated in Bear Stearns filing for bankruptcy on 13 March 2008 due to liquidity constraints. This triggered a sequence of events, fuelled by fears that financial institutions had excessive exposure to the sub-prime market, that would see the US Treasury providing guarantees for Fannie May and Freddie Mac, Bank of America acquire Merrill Lynch, the 100 year old Lehman Brothers file for chapter 11 bankruptcy and AIG lose more than 95% of their market value.

For an equity investor invested in indices such as the S&P 500 or MSCI, this meant a decline in more than half their wealth (from the highs of 2007 to the lows of 2009). Most governments were grappling to come to terms with the huge amounts of debt that were effectively transferred from the private sector to the public balance sheet, mostly due to guarantees made by governments in terms of returning stability to the global financial system.

South Africa, with its equity market closely linked to that of other international markets, such as the US and Europe, was not spared and suffered large investment losses. The All Share Index (ALSI) listed on the Johannesburg Stock Exchange (JSE) recorded a 40% drop in the index value from mid-2008 to early 2009. This was mainly due to the contagion effect and not instability within the South African banking environment.

After the 2008 market turmoil, asset management firms were either out of business or fighting for their existence. The asset management industry needed to reflect on their raison d'être as investors were increasingly critical of the fact that they were paying high active management fees, whilst only receiving benchmark-type returns. The asset management industry prides itself on being an industry driven solely by clients' needs and tailors solutions in an effort to satisfy these needs. However, through the years, the individual investor has been subjected to a one-size-fit-all type of solution, which is not always ideally constructed to fulfil the needs specified by the investor.

New trends that have emerged in the asset management industry attempt to expand the narrow focus of earlier days, where stock selection was recognised as the sole driver of returns. Nowadays, the value of asset allocation and portfolio construction is widely recognised in terms of building solutions to meet investor needs. The focus in the future will be to move towards dedicated portfolio solutions that address the problems that investors face, namely long-term consumption objectives in the presence of short-term constraints. This means that more focus will be placed on constructing enhanced benchmarks and employing advanced asset allocation strategies to solve identified investor difficulties, with less emphasis being placed on stock selection. To add value to the investor's wealth over time, active managers should improve on the beta returns that they achieve by using smart beta strategies to which alpha could be added as opposed to staying with traditional capital-weighted indices from which portfolios are constructed.

The ALSI, for example, is a traditional capital-weighted index for the JSE and that suffers from significant concentration risk. Measures such as the Herfindahl Hirschman Index[a] (HHI), which measures concentration, and the Portfolio Diversification Index[b] (PDI), which measures the diversity of the index, showed higher concentration (lower diversification) when compared to other indices, such as Share Weighted Index (SWIX). As the ALSI was heavily weighted to a few individual counters, the need to construct an index that would achieve increased diversification as one of its objectives (FTSE JSE SWIX[1], 2010) arose. This was achieved by constructing the SWIX index to reflect the investment universe for domestic funds, i.e. an index that is adjusted for foreign shareholding. Most asset managers in South Africa indicate the SWIX as the preferred benchmark from which to manage portfolios.

Table 1.1: ALSI vs. SWIX

30 June 2013	ALSI	SWIX
HHI	4.20	2.94
PDI	4.42	6.60
Top 10 holdings as % of index	56.42%	45.11%

Source: Factset

Analysis, however, suggests that the SWIX is too concentrated in a few large counters and therefore does not provide enough diversification to manage a well-diversified, risk-cognisant portfolio. For this purpose, we suggest a capped version of the SWIX. The newly constructed benchmark is based on the premise that a single stock should not be more than 8% of a portfolio as this would increase the concentration risk within the portfolio. In order for the active manager to demonstrate his preference for that stock, he/she should be able to overweight or underweight the stock based on his conviction. The capping would thus be set at 4%, which would enable the active manager to overweight the stock by 4% if he/she has high conviction, or have no holding in the stock, i.e. -4% underweight. We have tested this capping scheme against our measures of concentration and diversification and the results are shown in table 1.2.

Table 1.2: ALSI vs. SWIX vs. Capped SWIX

30 June 2013	ALSI	SWIX	Capped SWIX
HHI	4.20	2.94	2.07
PDI	4.42	6.60	10.17
Top 10 holdings as % of index	56.42%	45.11%	36.96%

Source: Factset

The results indicate that the Capped version of SWIX is more diverse when compared to the ALSI or SWIX.

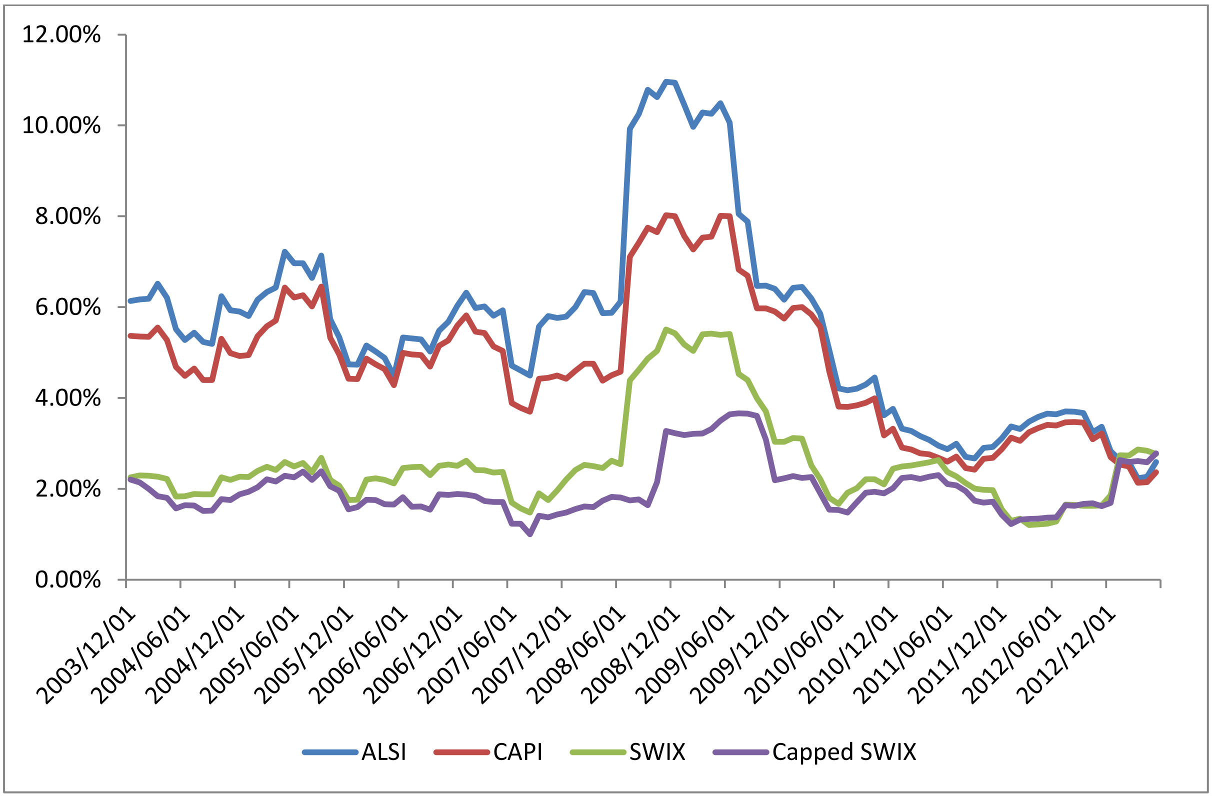
On closer inspection, it was also discovered that over time, large asset managers tend to track the Capped SWIX more closely than the SWIX when comparing the average equity return of the

[a] Herfindahl Hirschman index: Lower values indicate less concentration

[b] PDI: Higher values indicate more diversity

large managers to the SWIX, Capped SWIX, ALSI and CAPI (a capped version of the ALSI) and calculating the relevant realised tracking error (TE) of these returns to the indices (see figure 1.1).

Figure 1.1: Comparison of tracking error (TE)



Source: Alexander Forbes surveys

The question arises as to how to improve on these indices. Since Markowitz[2] (1952) introduced the concept of the efficient frontier, where an investor would choose a portfolio that maximised the expected return for a given level of risk (mean-variance optimisation), practitioners have struggled to incorporate these insights when building realistic portfolios. The challenge arises as achieving optimal diversification under the mean-variance assumption requires estimates for both the risk and return parameters. It has been shown that the Markowitz mean-variance optimisation is extremely sensitive to small changes in the expected return estimates (sometimes referred to as error maximisation and not return maximisation (Michaud[3], 1998)). The sensitivity of the input parameters has been illustrated by Best and Grauer[4] (1991), which demonstrates how half of the portfolio's assets changes when only one security's expected return changes by a small amount.

There is also the curse of dimensionality to consider. For N assets, one needs to estimate N expected returns, N variances and (N*(N-1))/2 correlations. For a portfolio consisting of 100 assets, this would imply estimating 100 expected returns, 100 variances and 5 000 correlations. This clearly introduces substantial estimation error potential. We therefore need to consider robust methods to estimate these parameters.

Let us first consider methods used to estimate the covariance matrix. In the EDHEC Survey[5] on Practitioner Portfolio Construction and Performance Measurement: Evidence from Europe (2011), 60% of participants indicated that they still use the sample covariance matrix in the portfolio construction process, 4% use a more sophisticated shrinkage approach and the rest used some type of factor model. As argued above, using the sample covariance matrix increases estimation error as the number of assets increase. Extreme elements within the covariance matrix skew the optimiser and the portfolio may be populated with some assets that have unnecessarily large weights.

In order to alleviate the effect that the extreme elements have on the mean-variance optimisation and reduce the estimation error from the sampling estimate, one can impose a structure. However, one needs to consider the trade-off between the sampling error associated with the sample covariance matrix and specification error that is introduced by a more structured approach to estimating the covariance matrix. Some of the more structured methods that introduce specification error include:

- the constant correlation approach (Elton and Gruber[6], 1973)
- the single factor forecast (Sharpe[7] 1963)
- multi-factor forecast (Chan, Karceski and Lakonishok[8] 1999).

There are also methods that attempt to combine the best of both worlds, i.e. reduce sampling error by introducing some structure within the covariance matrix. Some of these optimal structure methods include:

- shrinkage towards the constant correlation (Ledoit and Wolf[9] 2004)
- shrinkage towards the single factor model (Ledoit[10] 1999)
- portfolio constraints (Jagannathan and Ma[11] 2000).

Another method includes 'resampling efficiency', as suggested by Michaud[3] (1998) and reviewed and critiqued by Scherer[12] (2002). Wolf[13] (2006) compared the shrinkage approach vs. resampling and found that, although resampling improves on the estimates of the sample covariance, it is inferior to the shrinkage approach. Yet another approach could be to combine two sample covariance matrices as covariance matrices are additive. Chow[14] et al. (1999) suggested estimating a covariance matrix based on outliers as a proxy for a particular extreme event scenario (such as the financial crisis of 2008) and a traditional covariance matrix over a longer period. They propose blending these two matrices in such a manner as to reflect one's view of the risk environment and risk aversion. Combining these matrices in, say, a 20/80 proportion captures the long-run effect, as well as elements of the extreme scenario or downside risk.

The South African stock market is mostly driven by the RESI sector as the returns of these stocks are highly volatile and cluster together. Anglo American and BHP Billiton alone contribute around 20% of the market capitalisation. Based on historical trends, the South African stock market can be reduced to a view on either RESI (resource stocks) or FINDI (financial and industrial stocks). We acknowledge the fact that the sample covariance matrix introduces high estimation error as the resource stocks are volatile and could lead to extreme elements in the covariance matrix. An improvement in the estimation of the covariance matric would involve shrinking towards the constant correlation for the RESI and FINDI sectors.

The number of assets is also relatively small compared to other markets and we do not have the problem that the number of assets exceeds the number of observations and the matrix will therefore be invertible assuming the linear independence of the stocks. Amenc[15] et al. (2010) suggested a multi-factor approach to reduce the dimensionality of the covariance matrix; however, this is not necessary in our market.

Point estimates, such as the sample mean, are worthless and can't be used for reasons provided earlier. A structure could be imposed to estimate the expected return. Some of the structures that could be imposed are:

- single-factor models (CAPM)
- multi-factor models (Fama and French[16], APT)
- shrinkage towards a prior (Black-Litterman)
- relating downside measures to expected returns.

The CAPM model postulates that the stock's excess return is a function of the market's excess return. This is captured by beta in the following regression:

$$(R_i - R_f) = \beta_i * (R_m - R_f) + \text{error term}$$

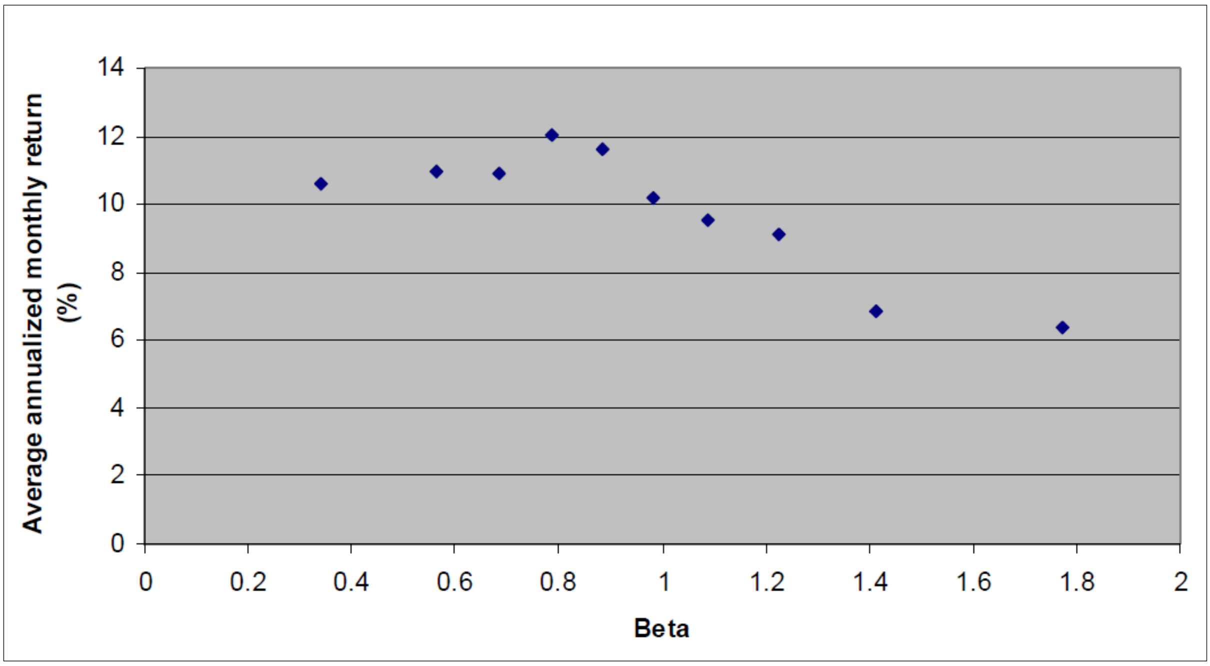
Where: R_i is the stock return, R_f is the risk-free rate, R_m is the market return and β_i is the sensitivity of the excess return of stock i to the excess return of the market.

This implies that stocks that tend to have a higher beta should have a higher return based on this model. Figure 1.2, however, illustrates that this is not always the case. The linear relationship between expected return and beta may break down; however, the study will employ betas as a measure for expected returns for the adjusted mean-variance approach

Note that the CAPM model implies that the intercept is zero and many studies have proved that the CAPM is not the true asset pricing model. Others, like Fama and French, have extended the single CAPM model to include additional factors, such as style, size and APT, even including economic and fundamental factors.

Another school of thought has opted for the use of a Bayesian approach, such as suggested by Black-Litterman. This method is well suited to an asset manager who has active views. These subjective views need to be reflected in the portfolio in a meaningful manner. According to Idzorek[17] (2005), the Black-Litterman model addresses the main problems associated with mean-variance optimisation, namely that portfolios are overly concentrated and that the optimisation routine is highly sensitive to the input parameters, which leads to large estimation errors. The Black-Litterman method uses the CAPM equilibrium market model as the prior or starting point to obtain the implied expected returns through the use of 'reverse optimisation' and blends this with the investor's views.

Figure 1.2: Average Annualised Monthly Return versus Beta for equally weighted portfolios (Russell 3000) Dec. 1986 – Dec. 2008



Source: SSgA, Barra

There is, however, another school of thought that has suggested that expected returns could be linked to downside risk. Both Zhang[18] (2005) and Boyer and Mitton and Vorkink[19] (2009) found that there was a positive relationship between expected returns and 'skewness'. Other authors, such as Tang and Shum[20] (2003), Connrad, Dittmar and Ghysels[21] (2009) also found a positive relationship between expected returns and skewness, but could not find the same for kurtosis. Ang[22] et al. (2006) concentrated on downside correlation and found a positive relationship with expected returns. Both Huang[23] et al (2009) and Bali and Cakici[24] (2004) used value at risk calculations based on extreme value theory and historical VaR to find a positive relationship to expected returns. Semi-deviation has also been found to show a positive relationship to expected returns, according to studies done by Chen[25] et al. (2009) and Estrada[26] (2000).

Different techniques used to estimate the covariance matrix and expected returns were illustrated. These parameter estimates are required as inputs into the mean-variance optimisation to solve the portfolio selection problem, however, test have been done to determine whether the performance of these optimal portfolios outperform techniques such as equal risk contribution, maximum diversification or the naïve 1/N diversified portfolio. A paper by DeMiguel[27] et al. (2009) evaluates the performance of different optimal portfolios using different optimisation techniques against the naïve 1/N portfolio. Two important findings of the study are worth noting: firstly, that more should be done to improve the estimates of asset returns and other relevant information on stock returns and, secondly, that the naïve 1/N portfolio should be the first benchmark to consider as a starting point. This is due to the result that the out-of-sample Sharpe-ratio of the mean variance portfolio is much lower than the naïve 1/N. The same input parameters used within the adjusted mean-variance portfolio would be used in the equal risk contribution and maximum diversity portfolios.

The rest of the paper will be structured as follows: Section 1.2 will focus on the data used to perform the analysis, while section 1.3 will describe the methodology employed to construct various smart beta benchmarks. In section 1.4, the results of the study will be evaluated for each of the smart beta strategies and section 1.5 will conclude.

1.2 Data

The study incorporates JSE-constituent data for the All Share Index for the period 2008 to 2012, but also makes use of weekly stock return data prior to 2008 to estimate covariance matrices in order to construct a smart beta benchmark from 2008. These benchmarks are rolled with market returns on a stock level for a quarter and rebalanced against its investment philosophy at the end of the quarter. Statistics such as return, risk and portfolio turnover form part of the result section.

1.3 Methodology

1.3.1 Naïve diversification

The study incorporates the simple 1/N diversified portfolio constructed from the constituents of the ALSI rebalanced quarterly where N equals the number of constituents present in the ALSI for that quarter. Each weight in the portfolio is calculated as:

$$w_{EW} = \frac{1}{N}\mathbf{1}$$

1.3.2 Simple mean-variance

To estimate the mean-variance portfolio, 5 years of weekly historical returns are used. The average weekly historical return for each stock is used as the expected return and the historic weekly covariance matrix is calculated for the preceding 5 year period. The portfolio will be rebalanced quarterly. Each weight in the portfolio is calculated as:

$$w_{MV} = \frac{\Sigma^{-1}\mu}{\mathbf{1}'\Sigma^{-1}\mu}$$

1.3.3 Adjusted mean-variance

The expected returns are based on the beta of the stock against the SWIX index as a proxy for the expected return. Some structure was introduced when estimating the covariance matrix. The average correlation of resource stocks (RESI) and financial and industrial stocks (FINDI), as well as the average cross correlation between them, was calculated. These estimates are combined with the correlation estimate for and between the other asset classes and the sample covariance matrix is shrunk optimally towards this structured covariance matrix, as per Ledoit and Wolf[9] (2004). The portfolio will be rebalanced quarterly. Each weight in the portfolio is calculated as:

$$w_{AMV} = \frac{\Sigma_A^{-1}\mu_A}{\mathbf{1}'\Sigma_A^{-1}\mu_A}$$

1.3.4 Simple Global Minimum variance

Global minimum variance optimisation requires no estimate of expected returns, which makes this type of optimisation popular and reduces the sensitivity associated with estimating expected return. Each weight in the portfolio is calculated as:

$$w_{GMV} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}$$

1.3.5 Adjusted Global Minimum variance

A structured approach similar to the one used within the adjusted mean-variance optimisation is employed to calculate the global minimum variance portfolio at the end of each quarter. Each weight in the portfolio is calculated as:

$$w_{AGMV} = \frac{\Sigma_A^{-1}\mathbf{1}}{\mathbf{1}'\Sigma_A^{-1}\mathbf{1}}$$

1.3.6 Maximum diversification

The Portfolio Diversification Index [a] (PDI) is used as the objective function and maximised at each quarter to obtain the maximum diversified portfolio. PDI is calculated as:

$$PDI = 2\sum_{k=1}^{N} k\lambda_k - 1$$

where N is the number of assets in the portfolio and λ_k is percentage contribution of factor k to total volatility. Each weight in the portfolio is adjusted to maximise the PDI of the portfolio.

1.3.7 Risk parity

The risk parity of equal risk contribution portfolio will also make use of the shrunk covariance matrix and optimised at the end of each quarter. In this portfolio each weight in the portfolio is adjusted such that:

$$w_i MCR_i = w_j MCR_j$$

where MCR is the marginal contribution to risk

1.3.8 ALSI, SWIX and Capped SWIX

These indices' characteristics will be compared to the other optimised portfolios.

1.4 Results

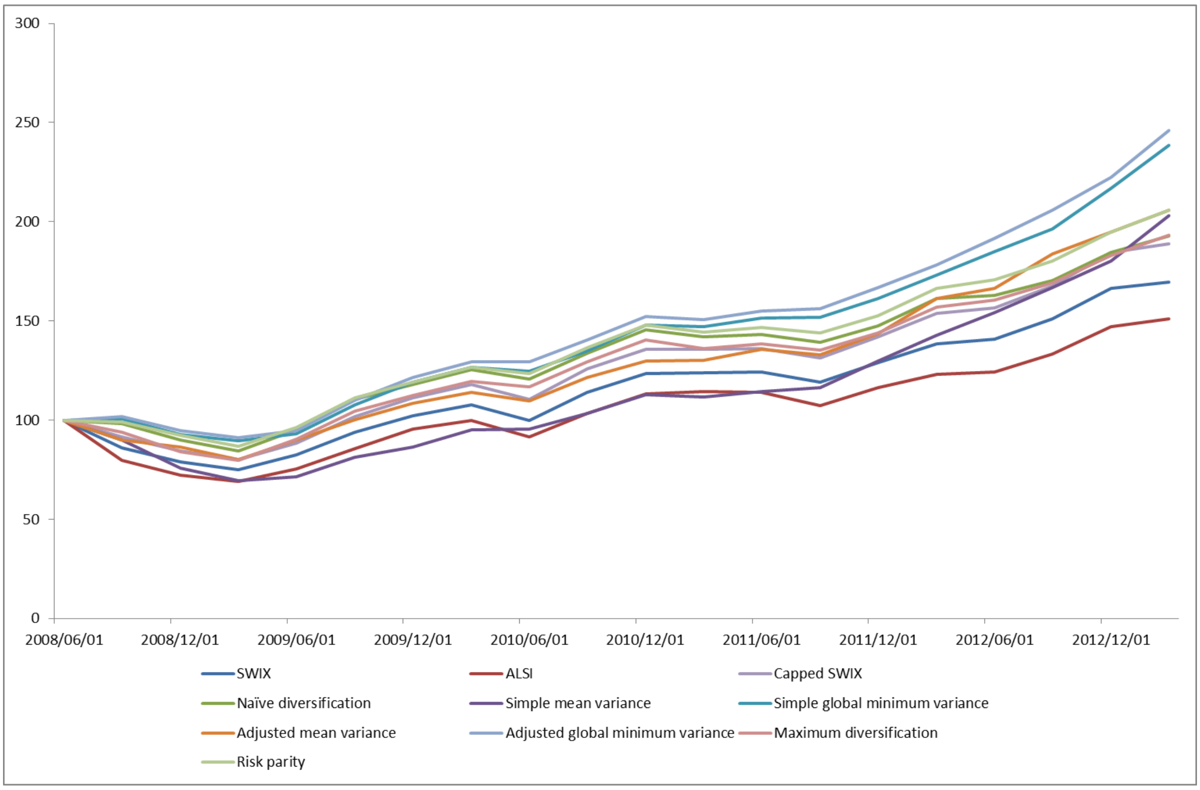
As summarised in table 1.3, all of the constructed portfolios outperformed the traditional capital weighted indices in terms of risk and return metrics. The portfolios constructed via structural adjustments to the covariance matrix and expected returns outperformed those that were not. Hence the adjusted mean variance and adjusted global minimum variance outperformed their simple counterparts in both risk and return measures. The maximum diversified portfolio outperformed the naïvely diversified portfolio in terms of annualised return, but had more risk. As expected, the risk parity portfolio fits in between the mean variance and global minimum variance portfolios in terms of risk and return. However, the adjusted mean variance portfolio generated a comparable Sharpe ratio to the risk parity portfolio with a larger annualised return.

Table 1.3: Summary analysis

	ALSI	SWIX	Capped SWIX	Naïve diversification	Simple mean variance	Adjusted mean variance	Simple global minimum variance	Adjusted global minimum variance	Maximum diversification	Risk parity
Annualised return	8.55%	11.23%	13.48%	13.87%	16.04%	16.34%	19.16%	20.04%	14.24%	15.56%
Annualised risk	35.12%	30.83%	28.97%	26.36%	31.67%	26.31%	22.52%	21.70%	27.63%	24.33%
Sharpe ratio	0.24	0.36	0.47	0.53	0.51	0.62	0.85	0.92	0.52	0.64

Shown in figure 1.3, the adjusted global minimum variance portfolio outperformed all other portfolios over most periods on a total return since inception basis, closely followed by the simple global minimum variance portfolio.

Figure 1.3: Total return over time



This is at least partly due to the result that drawdowns are less severe in the minimum variance portfolios. From the figure it is also apparent that the traditional ALSI and SWIX indices performed poorly over most periods compared to the constructed portfolios. Even though the simple mean variance portfolio ends up fourth based on total return over the period it underperformed all portfolios and indices until September 2011. This highlights an inherent problem with typical mean variance portfolios in that they are very sensitive to the expected returns vector and sample covariance matrix. Significantly better returns over time can be generated by adjusting the expected returns and sample covariance as shown in the graph of the adjusted mean variance portfolio. Even though the overall total return for the period ends up being fairly similar to the simple mean variance portfolio, the adjusted mean variance portfolio shows a more stable excess return profile over the ALSI and SWIX. Of interest is also the fact that the capped SWIX index performed very similar over time to the maximum diversified portfolio. Capping an index typically does increase its diversification and capped SWIX does indeed have a higher PDI than either the ALSI or the SWIX. It should be noted that the PDI of the maximum diversified portfolio is significantly higher than that of the capped SWIX and that the similarity in total return profile is not just due to increased diversification.

Viewing the returns on a rolling one year basis shows a different return profile than the one shown in figure 1.3. As shown in figure 1.4, on a rolling one year basis the simple mean variance portfolio outperformed in most periods, tentatively suggesting that the simple methodology employed in the construction of the covariance matrix and expected returns vector might be a better reflection of prevailing market conditions on a rolling 1 year basis. The increased rolling one year return for the simple mean variance portfolio is however at a higher volatility, decreasing the Sharpe ratio on a rolling basis. The results on a rolling one year basis is summarised in table 1.4

Figure 1.4: Rolling 1 year return

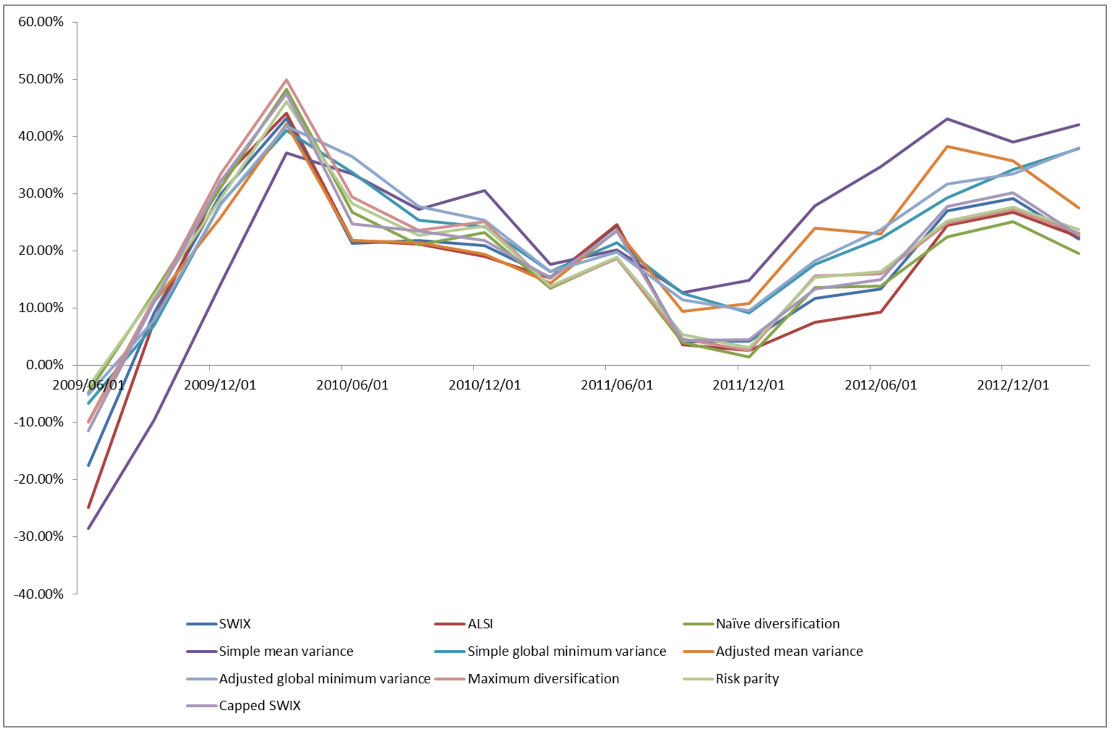


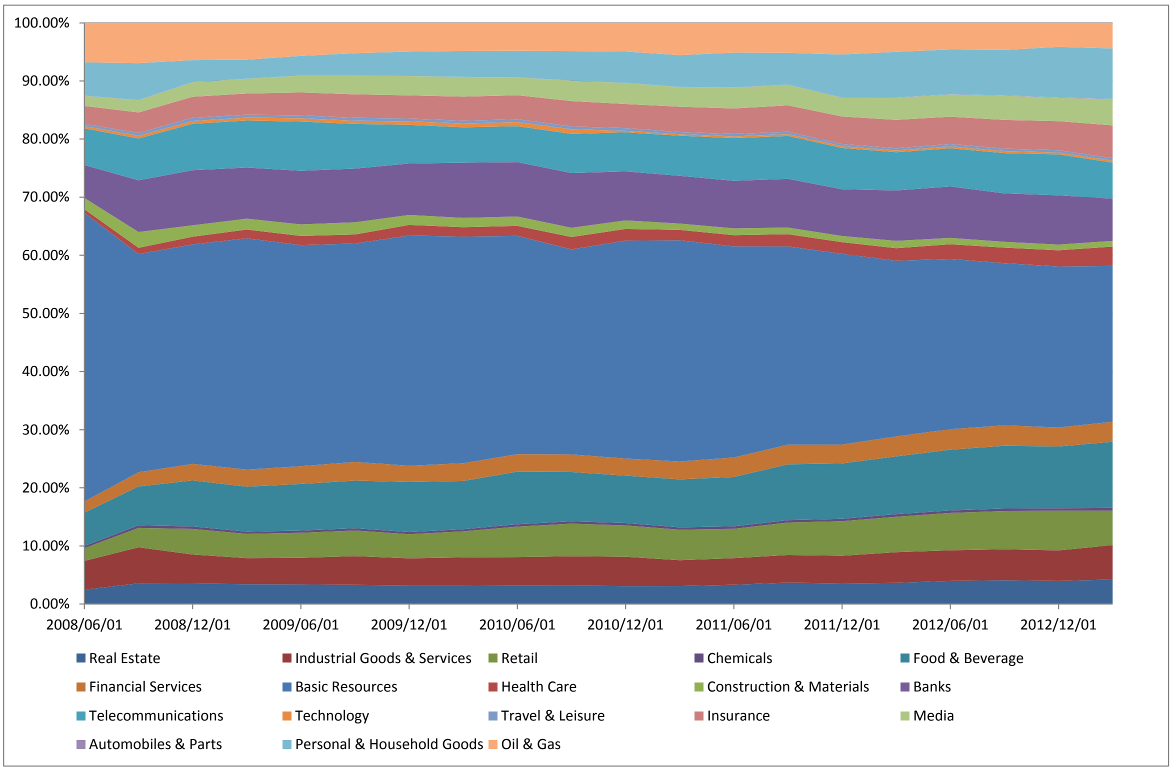
Table 1.4: Summary analysis – Rolling 1 Year

	ALSI	SWIX	Capped SWIX	Naïve diversification	Simple mean variance	Adjusted mean variance	Simple global minimum variance	Adjusted global minimum variance	Maximum diversification	Risk parity
avg rolling return	16.08%	17.49%	19.11%	18.14%	22.27%	21.12%	22.10%	22.76%	19.32%	19.27%
avg rolling risk	15.52%	13.81%	13.58%	12.53%	19.25%	12.63%	12.54%	12.69%	13.80%	11.97%
Sharpe ratio	1.04	1.27	1.41	1.45	1.16	1.67	1.76	1.79	1.40	1.61

Once again the capped SWIX index performed very similarly to the maximum diversified portfolio. From table 1.4 it is clear that even though the simple mean variance portfolio generates a fairly high average rolling one year return, it does so at significant volatility, resulting in a Sharpe ratio less than the SWIX. On a risk adjusted basis the top performing portfolio on a rolling one year basis was once again the adjusted global minimum variance portfolio followed by the simple global minimum variance portfolio at Sharpe ratios of 1.79 and 1.76 respectively.

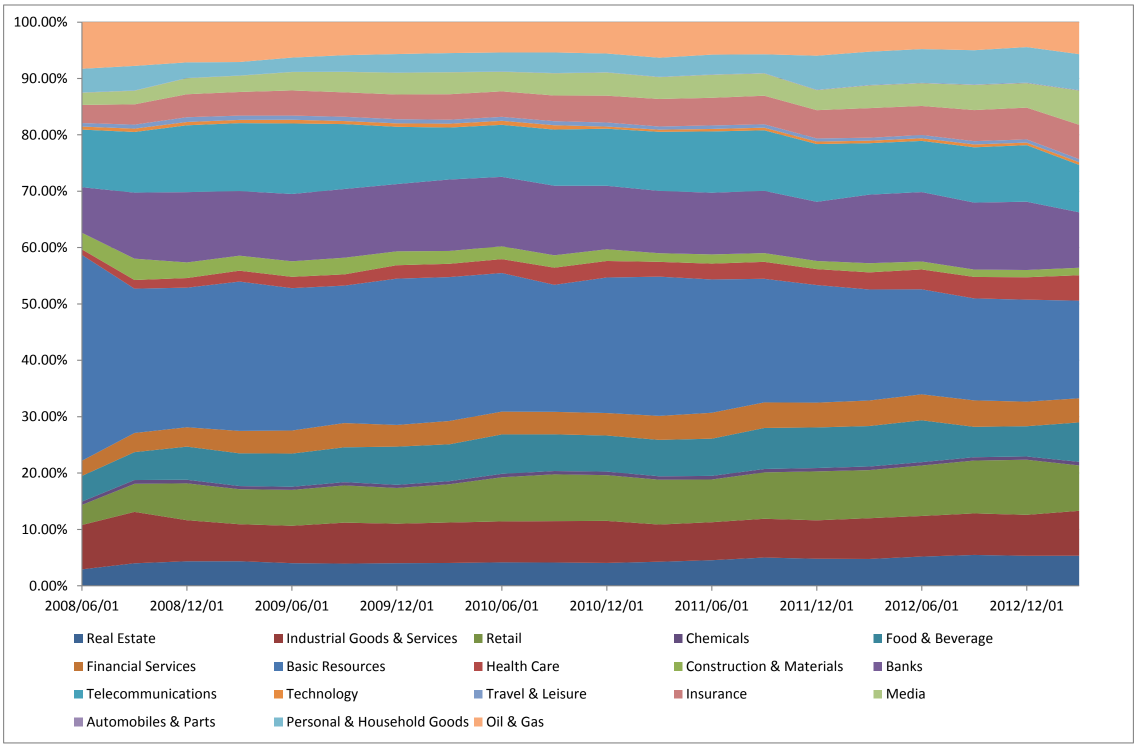
Each index or portfolio has inherent structural biases associated with it. From figure 1.5 it is clear that the ALSI has a significant concentration in the basic resources sector. On average nearly 36% of the ALSI is based in the basic resources sector.

Figure 1.5: ALSI sector weights over time



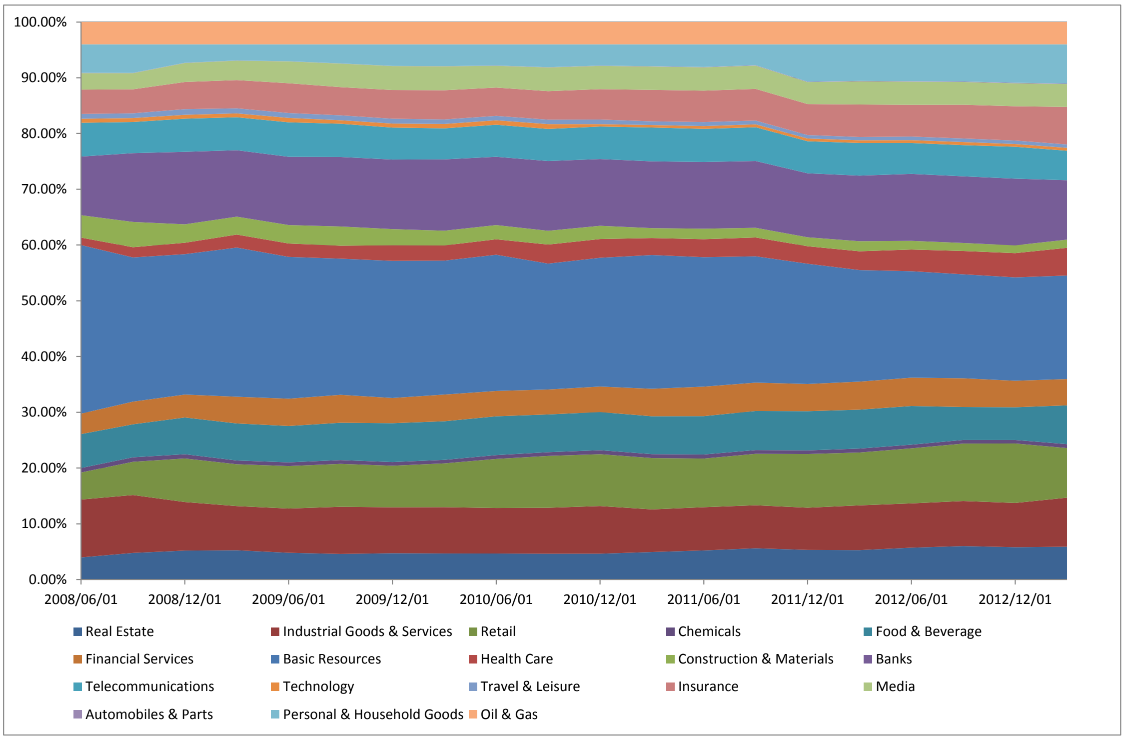
A similar profile is generated for the SWIX in figure 1.6, although the average weight in the basic resources sector is now down weighted to 23% whilst the telecommunications sector has been substantially up weighted, due to MTN's weighting in SWIX.

Figure 1.6: SWIX sector weights over time



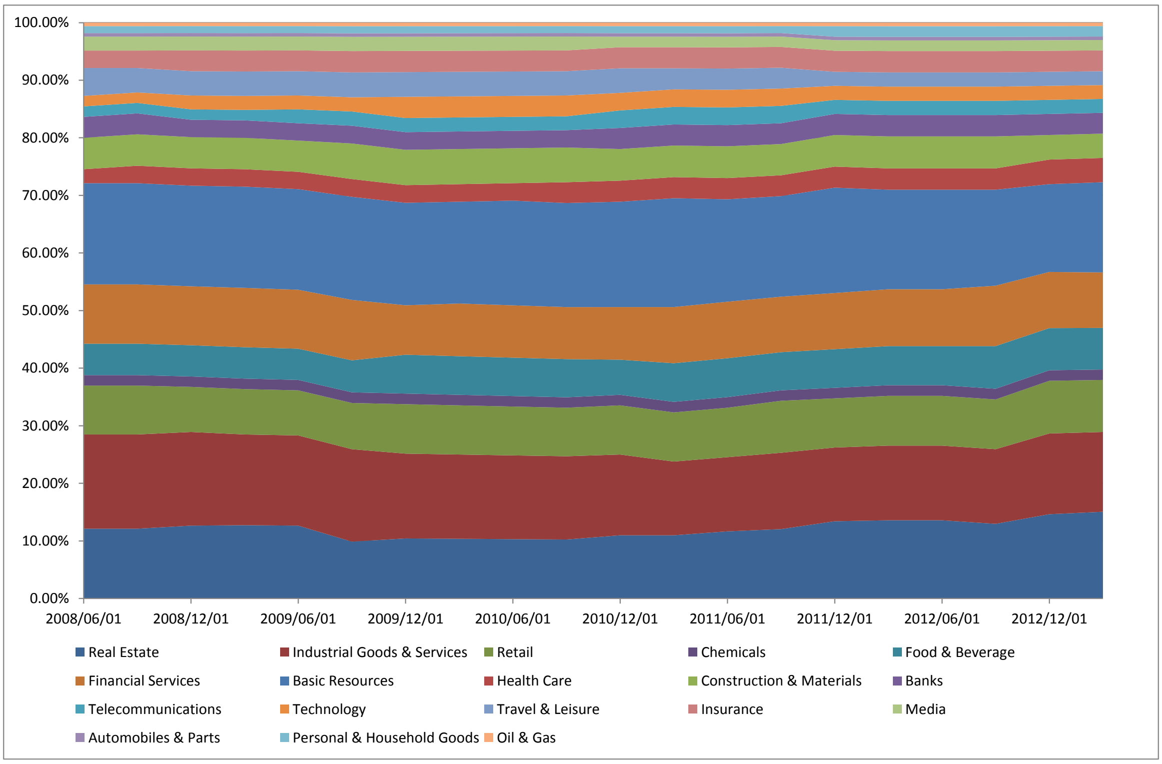
Capping the SWIX further reduces the concentration of any single sector in the index as shown in figure 1.7. Concentration risk in MTN is also reduced due to the SWIX capping.

Figure 1.7: Capped SWIX sector weights over time



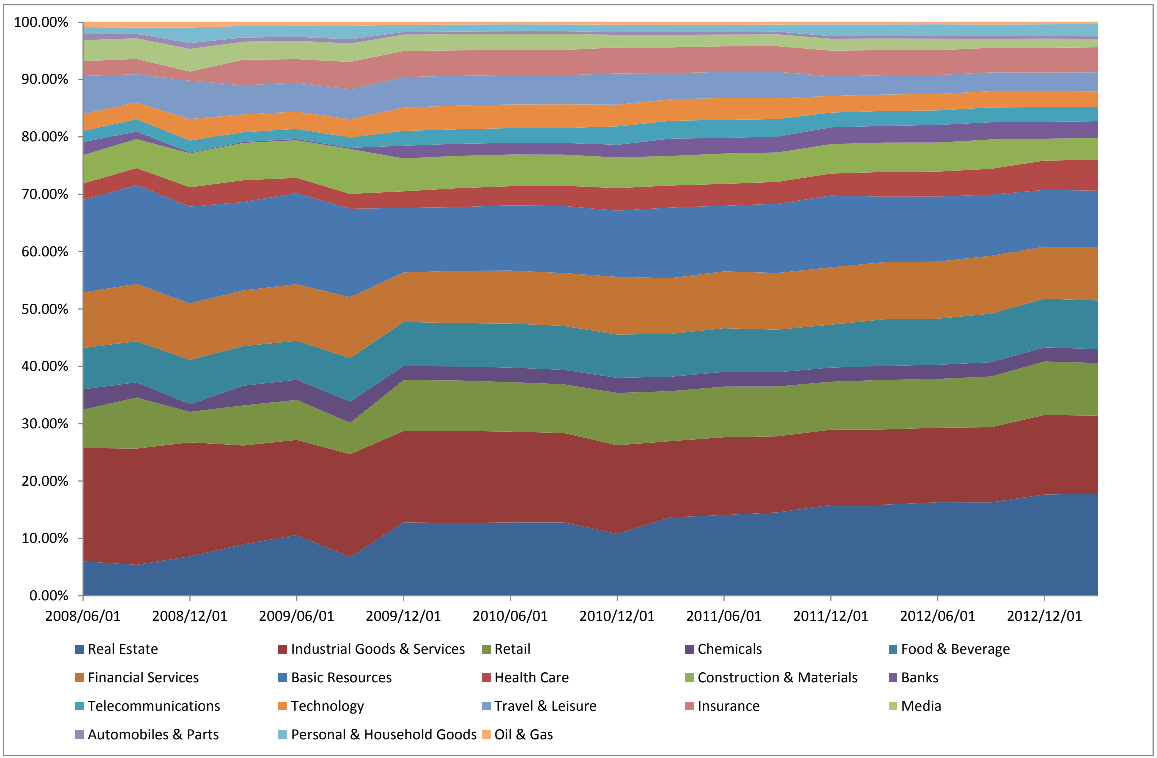
As expected the naïve diversified portfolio has a fairly stable sector weight contribution over time, as shown in figure 1.8, with only 18% of the portfolio on average in basic resources. Due to the number of real estate companies in the index though, the naïve diversified portfolio does however have a much larger weight in the real estate sector when compared to ALSI, SWIX or capped SWIX (12% versus 3.5%, 4.4% and 5% respectively)

Figure 1.8: Naïve diversified portfolio sector weights over time



A similar graph is obtained when examining the maximum diversified portfolio, as shown in figure 1.9. The slightly more irregular sector bands indicate that in order to maximise diversification the portfolio up weights and down weights sectors on occasion. The basic resources sector is only 13% in the maximum diversified portfolio, as is real estate, whilst the single biggest sector in the index is the industrial goods & services sector at 15%. The irregularity of the sector bands also indicates that portfolio turnover is higher than would be the case in the naïve diversified portfolio.

Figure 1.9: Maximum diversified portfolio sector weights over time



The sector weight distribution of the simple mean variance portfolio in figure 1.10 shows significant up weighting and down weighting of sectors. Here basic resources constitute a mere 6.5% of the portfolio on average and the real estate sector constitutes 20% of the portfolio. The same is not true for the adjusted mean variance portfolio, as seen in figure 1.11.

Figure 1.10: Simple mean variance portfolio sector weights over time

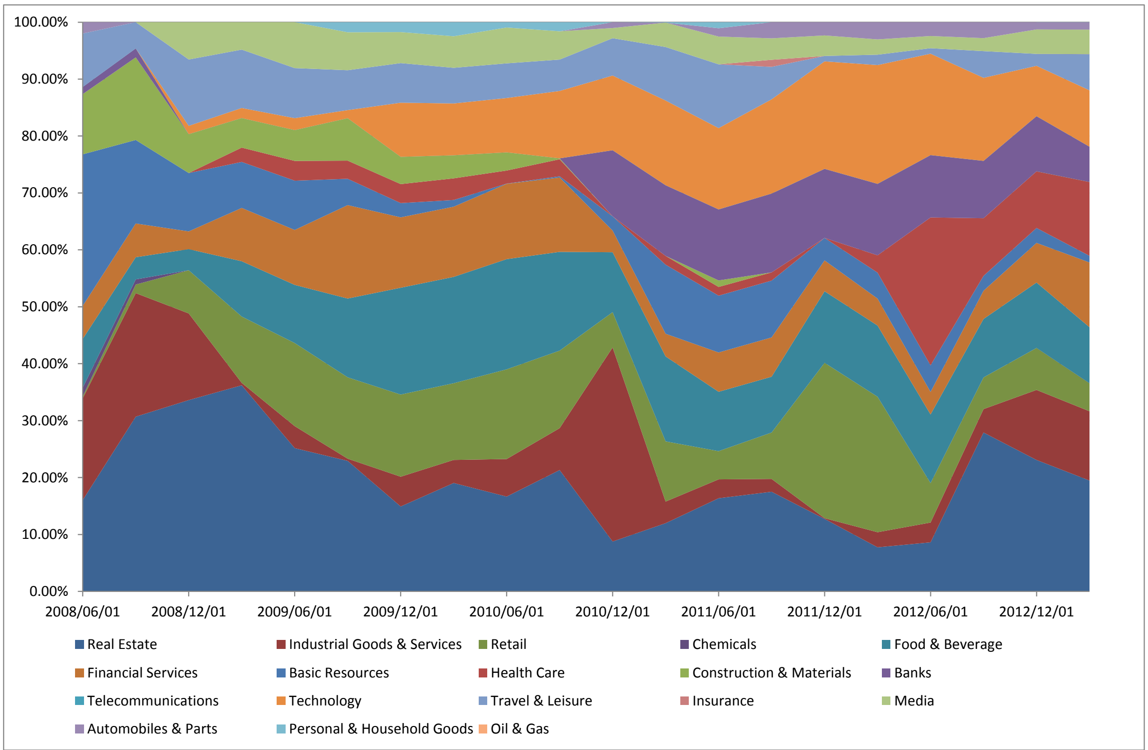
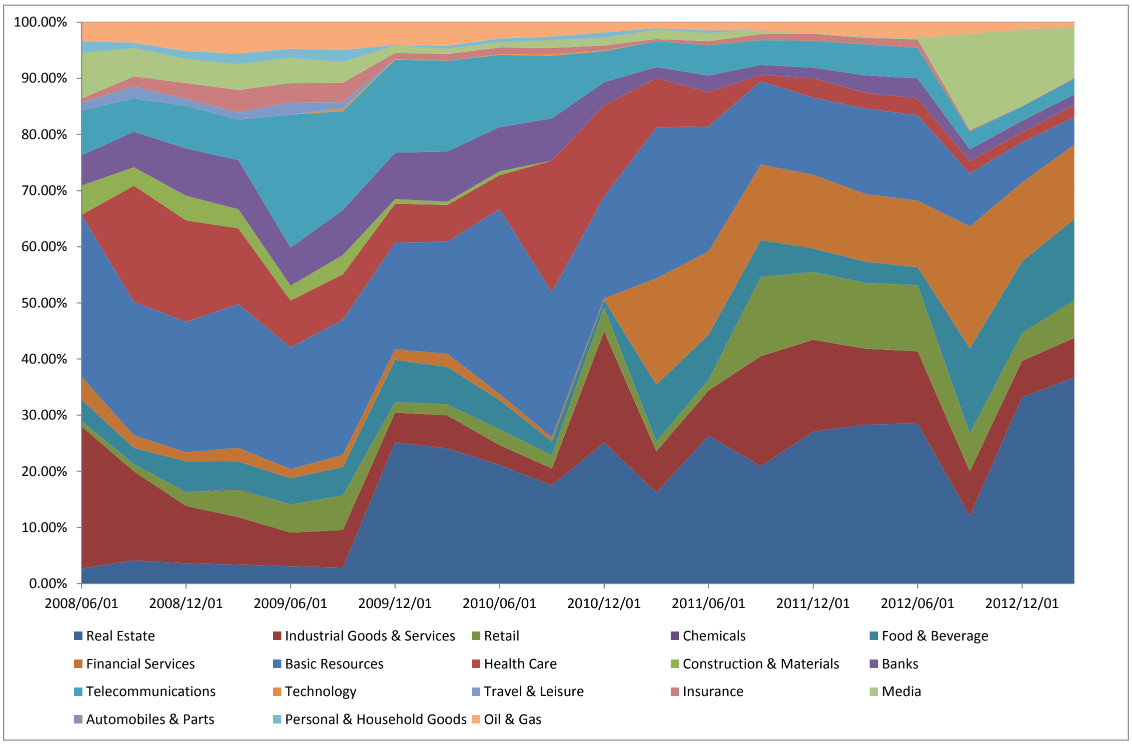


Figure 1.11: Adjusted mean variance portfolio sector weights over time



In the adjusted mean variance portfolio there is once again significant up weighting of the real estate sector, averaging 18% of the portfolio for the period. The basic resource sector is however not as severely down weighted and makes up nearly 20% of the portfolio on average.

Due to the relatively low volatility of the real estate sector, it is not surprising that the simple global minimum variance portfolio and the adjusted global minimum variance portfolio have significant weightings in it. In the case of the simple global minimum variance portfolio, real estate constitutes 22% of the portfolio, whereas in the case of the adjusted global minimum variance portfolio it is 25% on average, going as high as 32% on occasion. Neither portfolio has any significant weight in the basic resources sector due to the preference of low volatility sectors. The initial preference for financial services in both portfolios is largely explained by a number of small, relatively illiquid, financial services stocks that exhibited an artificially low volatility for the period.

Finally the risk parity portfolio shows a much more stable sector weighting over time. As seen in figure 1.14 the sector weights are much more evenly distributed, with the main sectors being real estate, industrial goods & services and basic resources at 15.5%, 14.7% and 13% on average respectively. The sector weighting profile of the risk parity portfolio resembles both the maximum diversified and naïve diversified portfolios.

As is suggested from the sector composition graphs, the various portfolios and indices turn over significantly different amounts of the portfolio on a quarterly basis. The average quarterly turn over for the various portfolios is summarised in table 1.5.

Figure 1.12: Simple global minimum variance portfolio sector weights over time

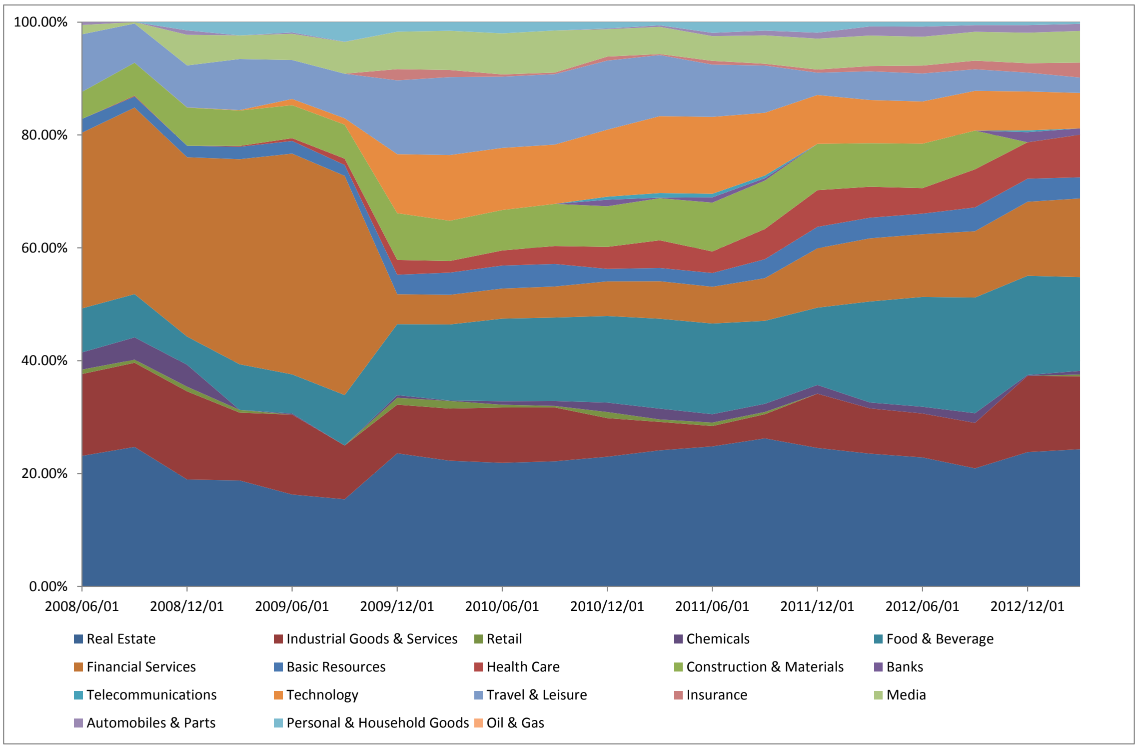


Figure 1.13: Adjusted global minimum variance portfolio sector weights over time

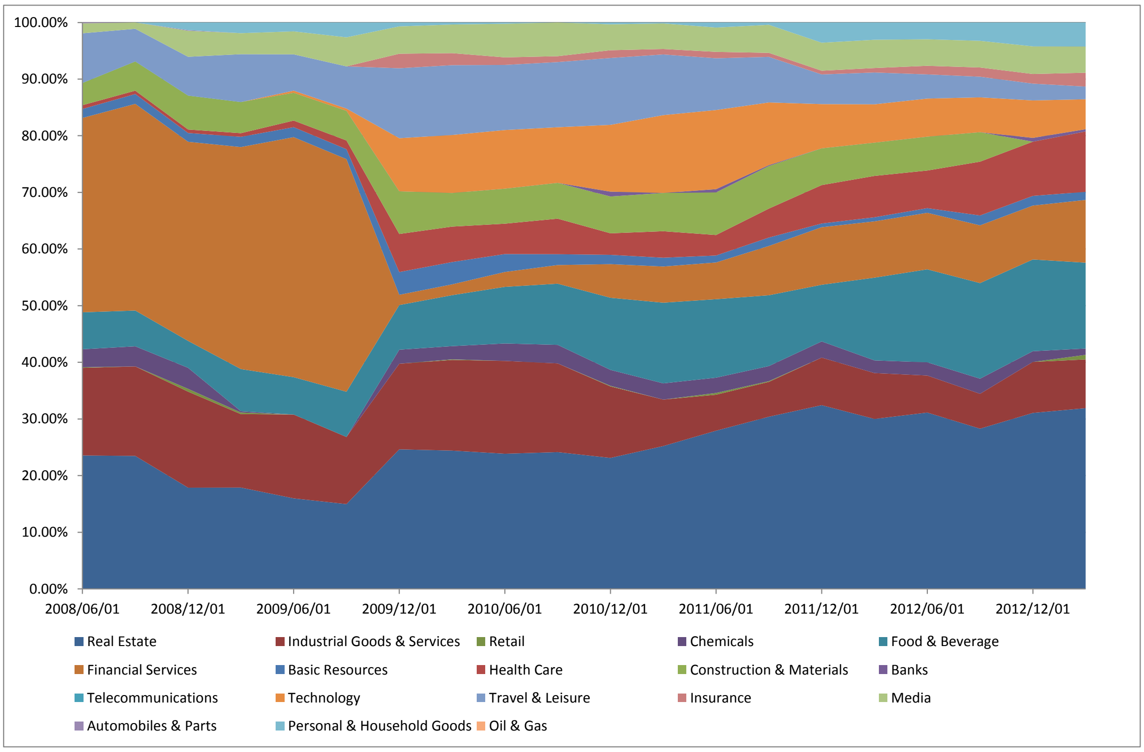


Figure 1.14: Risk parity portfolio sector weights over time

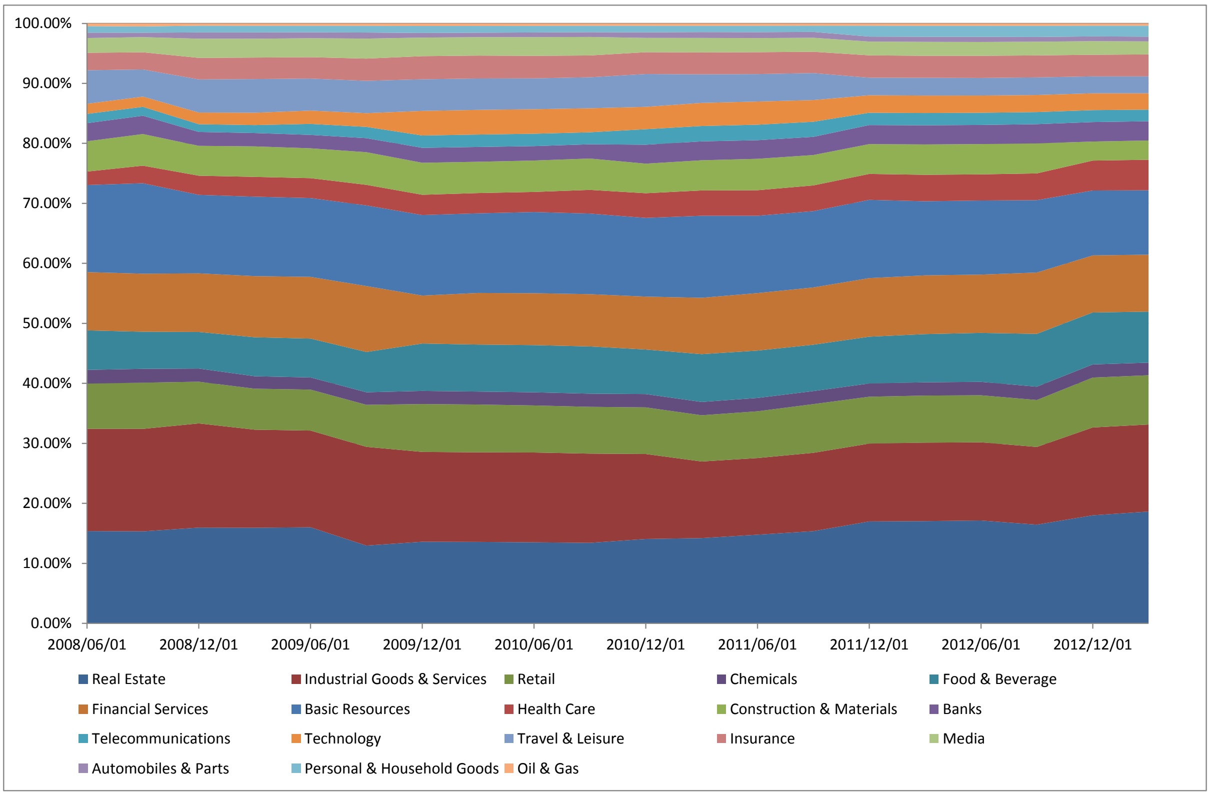


Table 1.5: Average quarterly turn over statistics

	Average turn over
ALSI	2.76%
SWIX	4.26%
Capped SWIX	6.23%
Naïve diversification	12.04%
Simple mean variance	52.57%
Simple global minimum variance	28.73%
Adjusted mean variance	49.52%
Adjusted global minimum variance	26.23%
Maximum diversification	22.82%
Risk parity	12.08%

The traditional capital weighted indices and the capped version thereof has significantly less quarterly turn over than any of the constructed portfolios. The two mean variance portfolios are especially prone to a very high quarterly turn over. Of interest is the slight reduction in average turnover of the simple mean variance and simple global minimum variance portfolios to their adjusted counterparts. This suggests that making the structural changes to the covariance matrix and expected return vector can introduce a more stable weighting scheme within the portfolio leading to decreased turnover (although the effect is slight in this instance). Depending on trading costs and other considerations during the investment process (such as income versus capital gains tax) the high turnover for some of the constructed portfolios may exclude them as viable investment strategies in certain instances.

Trading costs due to turnover do not substantially impact the overall risk/return characteristics of the various indices and constructed portfolios. Table 1.6 shows the impact of trading on the risk/return metrics, assuming trading costs of 0.50%. The Sharpe ratio is reduced in most instances, but the two global minimum variance portfolios still outperformed.

Table 1.5: Average quarterly turn over statistics

	ALSI	SWIX	Capped SWIX	Naïve diversification	Simple mean variance	Adjusted mean variance	Simple global minimum variance	Adjusted global minimum variance	Maximum diversification	Risk parity
Annualised return	8.51%	11.23%	13.33%	13.59%	14.83%	15.19%	18.47%	19.41%	13.72%	15.28%
Annualised risk	35.09%	30.77%	28.99%	26.37%	31.77%	26.42%	22.54%	21.71%	27.67%	24.34%
Sharpe ratio	0.24	0.36	0.46	0.52	0.47	0.57	0.82	0.89	0.50	0.63

The liquidity of the stocks is another aspect to consider when evaluating the feasibility of investing in these indices or constructed portfolios. The traditional capital weighted indices have a structural bias to large cap stocks, with the top 40 stocks by market capitalisation constituting more than 80% of the ALSI. Likewise large cap stocks make up nearly 75% of SWIX and 73% of capped SWIX. The naïvely diversified portfolio, maximum diversified portfolio and risk parity portfolios all have a fairly even distribution between large, medium and small cap stocks, with each group constituting between 20% to 40%.

The simple mean variance, adjusted mean variance, simple global minimum variance and adjusted global minimum variance portfolios all have a large structural bias to small cap stocks. It is somewhat mitigated by the structural changes to the covariance matrix, in that the adjusted mean variance portfolio favours small cap stocks less than the simple mean variance portfolio. Similarly, the adjusted global minimum variance favours small cap stocks less than the simple global minimum variance portfolios. Still, these portfolios all have a weighting in small cap stocks in excess of 50% which would make these portfolios unfeasible to invest in for large institutional investors.

Naturally, it is possible to adjust for these liquidity constraints by adjusting the initial investable universe when constructing the various optimised portfolios.

1.5 Conclusion

By making relatively simple adjustments to the traditional capital weighted indices, it is possible to generate superior returns at a lower volatility. Also, by introducing structural changes to the input parameters of a particular strategy (such as the covariance matrix or expected returns vector) it is possible to further enhance returns at even lower risk.

All indices (constructed or otherwise) exhibited certain structural biases, of which the investor must remain cognisant, in order to fully assess the behaviour of the index under various market conditions. The structural biases within these portfolios could be due to a concentration in certain sectors or favouring smaller, less liquid stocks.

The benefits of adjusting the traditional capital weighted indices were shown in terms of risk and returns, however, active managers and clients should be aware of the structural risks inherent in each of the weighting schemes. When investing, one is constantly reminded of the cliché about not putting all your eggs in one basket. During normal market activity, investors enjoy the 'free lunch' called diversification but, in the event of market extremes as experienced during 2008, this effect is reduced as risky assets tend to move in the same direction. Investors should realise that they are investing in inherently risky assets, and that as such it is important to stay invested over longer time horizons as well as to diversify between stocks and asset classes in order to appropriately reap the associated benefits. This also holds true for the constructed portfolios as most of these incorporate the benefits of diversification to ultimately outperform the traditional capital market weighted indices.

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